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NONLINEAR MATHEMATICAL MODEL OF DEMAND – SUPPLY

НЕЛІНІЙНА МАТЕМАТИЧНА МОДЕЛЬ ПОПИТУ-ПРОПОЗИЦІЇ

Currently, mathematical modeling is more and more persistently invading the economy. The possibility of using mathematical modeling is related to the existence of stable trends that characterize many economic processes. The importance of modeling as a research method is determined by the fact that the model is a conceptual tool focused on the analysis of the researched processes and their forecasting. The need to master mathematical modeling of economic processes as a method of analysis is not limited to purely practical needs: mastering this method contributes to the formation of non-linear thinking. Thus, in addition to solving purely practical problems, the use of this method has a great ideological significance. The paper examines two models of supply and demand: linear and nonlinear. For each model, goals and mathematical formulations of the models are formulated. Methods for analyzing the coefficients of two models are proposed.

Key words: *linear model, non-linear model, demand, minimum price, supply.*

В даний час математичне моделювання все наполегливіше вторгається в економіку. Можливість використання математичного моделювання пов'язані з існуванням стійких тенденцій, які характеризують багато економічних процесів. Значення моделювання як методу досліджень визначається тим, що модель є концептуальним інструментом, орієнтованим на аналіз досліджуваних процесів та їх прогнозування. Необхідність освоєння математичного моделювання економічних процесів як методу аналізу не обмежується суто практичними потребами: володіння цим методом сприяє формуванню нелінійного мислення. Таким чином, крім вирішення суто практичних завдань, використання цього методу має велике світоглядне значення.Метою дослідницької роботи є виклад та аналіз базових моделей економічних процесів. Розглядається лінійна модель попиту-пропозиції, тобто попит та пропозиція залежать від ціни лінійно. Отримано опис поведінки цін найближчими роками як функцію від первісної ціни. Результати досліджень показують, що ринок влаштований набагато складніше, і для нього потрібно вигадати іншу модель. Потрібно врахувати, що пропозиція неспроможна зростати вічно, оскільки продукцію неможливо створювати нескінченно багато, попит і пропозицію у якийсь момент гостріше реагують зміну ціни. Розглянуто нелінійну модель попиту-пропозиції, тобто попит та пропозицію залежать від ціни як складніші функції, ніж лінійна. Виробник щороку робить товар на продаж. Товар не зберігає більше року. Рішення у тому, скільки товару виробляти, приймається з урахуванням цін попереднього року. Причому, якщо ціни були високі – цього року треба випускати товару більше, а якщо низькі – менше. Попит товару протягом року залежить з його ціни на момент продажу. Коли ціна зростає, попит *падає. Модель описує поведінку цін найближчими роками як функцію від первісної ціни. Було досліджено дві моделі: лінійну та нелінійну, внаслідок чого отримано результати аналізу коефіцієнтів. Слід зазначити, що період розрахунку цін не може бути занадто великим, оскільки протягом великого часу коефіцієнти можуть змінитися. Тим самим отримано готові до використання формули зростання-падіння цін.*

Ключові слова: *лінійна модель,нелінійна модель, попит, мінімальна ціна, пропозиція.*

Formulation of the problem. Demand is the relationship between price and the quantity of a product that buyers can and are willing to buy at a strictly defined price in a certain period of time. Supply is a concept that reflects the behavior of a commodity producer on the market, his willingness to produce (offer) any quantity of goods over a certain period of time under certain conditions. Supply and demand depend on each other, so it is very difficult to predict them, but it can be calculated.

Analysis of recent research and publications. Socio-economic forecasting of the main directions of social development involves the use of special computational and logical techniques that make it possible to determine the functioning parameters of individual elements of the productive forces in their interrelation and interdependence. Systematized scientifically based forecasting of the development of socio-economic processes on the basis of specialized ones has been carried out since the first half of the 50s, although some forecasting techniques were known earlier. These include: logical analysis and analogy, extrapolation of trends, polling the opinions of specialists and scientists. In the development of the methodology for forecasting socio-economic processes, scientific developments of domestic and foreign scientists played a major role [1; 2]. The works of these scientists examine the meaning, essence and functions of forecasting, its role and place in the planning system, explore issues of methodology and organization of economic forecasting, and show the features of scientific forecasting. The development of works covering forecasting issues is carried out in the following main directions: deepening the theoretical and applied developments of several groups of techniques that meet the requirements of different objects and different types of forecasting work; development and implementation in practice of special methods and procedures for using various methodological techniques in the course of a specific forecast study; searching for ways and means of algorithmizing forecasting methods and implementing them using a computer [3]. According to estimates of domestic and foreign scientists, there are currently over 20 forecasting methods, but the number of basic ones is much smaller (15–20). Many of these methods refer rather to individual techniques and procedures that take into account the nuances of the forecast object. Others are a set of individual techniques that differ from the basic ones or from each other in the number of private techniques and the sequence of their application. Modeling nonlinear functions and propositions in economics is a key aspect for understanding and forecasting market intelligence. The broadest form of modeling is curved popit and proposition. They represent the relationships between the price of a product and the product that is sold or exchanged. Nonlinear models of these curves can include quadratic, exponential, logarithmic, or other functions to briefly describe the changes in the statement and proposition when the price changes. Another approach is to model the reaction and variation functions, which describe how people and companies react to changes in prices. These functions may also be non-linear, and there are non-linear relationships between income, price and income. Models of nonlinear functions and propositions also lack elasticity, which indicates the reaction of prices or propositions to price changes. These models may contain additional parameters that describe the behavior of consumers and producers in different scenarios. To take into account temporal changes in demand and supply, choose dynamic models. These models can describe changes in market conditions over time and increase the dynamics of reactions of subjects to market price changes. The growing interest in artificial intelligence, machine learning and big data analysis is improving the development of new innovative approaches in the modeling of non-linear supply and demand functions, which allows for more precise and accurate predictions of market conditions. These models and approaches help economists, analysts and business leaders better understand and analyze market processes to make better strategic decisions and predict changes in the economy [4; 5].

Formulation of the purpose of the article. The purpose of my research work is to present and analyze basic models of economic processes.

Presentation of the main material. *Linear supply-demand model.* Let's consider a linear supply-demand model, that is, supply and demand depend linearly on price [1; 2].

Meaningful formulation of the problem. The manufacturer makes a product for sale every year. He does not store the goods for more than a year. The decision on how much of a good to produce is made taking into account the previous year's prices. Moreover, if prices were high, more goods should be produced this year, and if prices were low, less. The demand for a product during the year depends on its price at the time of sale. When the price rises, demand falls. It is necessary to describe the behavior of prices in the coming years as a function of the initial price.

Conceptual statement of the problem. We use the following as model parameters: p_n – price per unit of goods in the n th year; s_n – supply (volume of supplies) of goods in the n th year; d_n – demand for the product in the n th year. We will build the model under the following assumptions:

The object of the study is the dependence of the price p_n of a product on its initial price p_0.

Suppose that next year's supply s_{n+1} depends linearly on the price p_n this year, and the higher p_n, the greater s_{n+1}: $s_{n+1} = ap_n - b$, where a and b are positive constants that remain unchanged throughout the entire analyzed period of time [1; 2]. Obviously, the price of a product should not be less than a certain minimum value that covers the costs of its production, only in this case the supply value s_{n+1} will be greater than zero.

Let us assume that next year's demand d_{n+1} depends linearly on the price p_{n+1} in the same year, and the higher the price p_{n+1}, the lower the demand d_{n+1}: $d_{n+1} = -cp_{n+1} + g$, where c and g are positive constants that remain unchanged throughout the entire analyzed time period. Obviously, the greatest demand for a product should exist at $p_{n+1} = 0$.

Let us assume that the market price p_{n+1} is determined by the equilibrium between demand d_{n+1} and supply s_{n+1}.

It is required to describe the behavior of prices p_1, p_2, p_3, ... depending on the value of price p_0.

Mathematical formulation of the problem. Assuming the value p_0 is given, find a sequence of values p_1, p_2, p_3, ...that satisfies the following system of equations:

$$s_{n+1} = ap_n - b, \tag{1}$$

$$d_{n+1} = -cp_{n+1} = g, \tag{2}$$

$$s_{n+1} = d_{n+1}, \tag{3}$$

where a, b, c, g are positive real numbers, and the ratios b/a and g/c characterize, respectively, the minimum and maximum permissible prices, and the value of g is the maximum possible demand (Fig. 1).

Methodology for solving the problem. Substituting equations (1) and (2) into equation (3), as well as making substitutions: $A = a/c > 0$, $B = \left(b/c + g/c\right) > 0$, we got the equation:

$$p_{n+1} = -Ap_n + B. \tag{4}$$

Equation (4) represents a linear recurrence relation that allows us to construct a sequence of solutions p_1, p_2, p_3, ...that are of interest to us. Let us rewrite (4) in the form

$$p_{n+1} + Ap_n = B. \tag{5}$$

Since we are interested in the functional dependence $p_n\left(p_0\right)$, consider the following solution scheme. We will look for the nth solution in the form of the sum of the solution to the homogeneous equation

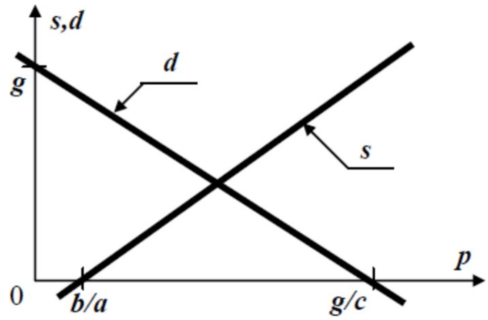

Figure 1. The values b/a and g/c are the minimum and maximum allowable prices, g is the maximum possible demand

Source: compiled by the author

$$p_{n+1} + Ap_n = 0 \tag{6}$$

and a particular solution to equation (5).

Solution of the homogeneous equation: $p_{n+1} = -Ap_n$. Let us assume that $p_0 = C$. Then $p_1 = C \cdot (-A)$; $p_2 = C \cdot (-A)^2$; $p_3 = C \cdot (-A)^3$; …… or in general $p_n = C \cdot (-A)^n$. Particular solution of the inhomogeneous equation: based on the form of the right-hand side of (5), we will look for a solution in the form of a constant $p_n = D$ for all n. Substituting into (5), we get $D + AD = B$ or $D = B/(A+1)$. Therefore, the general solution (5) has the form

$$p_n = C \cdot (-A)^n + B/(A+1). \tag{7}$$

From (7) with $n = 0$ we obtain

$$C = p_0 - B/(A+1). \tag{8}$$

Let us substitute (8) into (7) and finally obtain a solution to the problem in the form:

$$p_n = p_0(-A)^n + \left[B/(A+1)\right] \cdot \left[1 - (-A)^n\right]. \tag{9}$$

Analysis of results. By condition $A > 0$. From consideration of relation (9), three characteristic ranges of values of A can be distinguished:

1) When $0 < A < 1$ the market turns out to be balanced (price p_n tends to $B/(A+1)$) (Fig. 2).

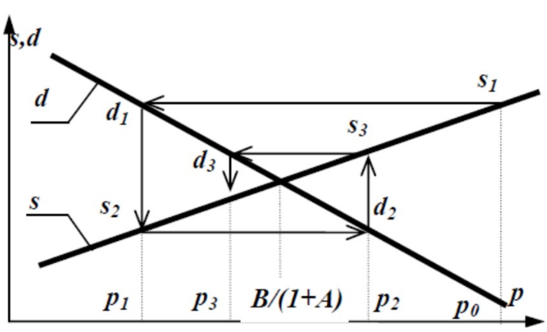

Figure 2. Balanced market at $0 < A < 1$

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2) When $A=1$, the market turns out to be unstable (there is a periodic decrease and increase in price) (Fig. 3).

3) When $A>1$, the market turns out to be completely unstable (collapse) (as n increases, the amplitude of fluctuations pn increases) (Fig. 4).

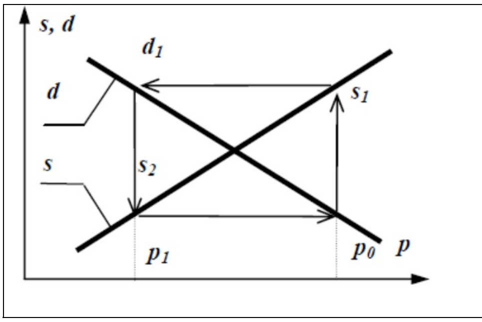

Figure 3. When $A=1$ there is a periodic decrease and increase in price

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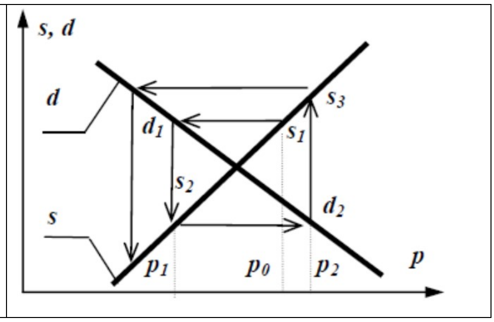

Figure 4. When A>1 the market turns out to be completely unstable

Source: compiled by the author

Nonlinear supply-demand model. That is, supply and demand depend on price as functions that are more complex than linear [1–2].

Meaningful formulation of the problem. The manufacturer makes a product for sale every year. He does not store the goods for more than a year. The decision on how much of a good to produce is made taking into account the prices of the previous year. Moreover, if prices were high, more goods should be produced this year, and if prices were low, less. The demand for a product during the year depends on its price at the time of sale. When the price rises, demand falls. It is necessary to describe the behavior of prices in the coming years as a function of the initial price.

Conceptual statement of the problem. As model parameters we use parameters from the linear model with the exception of the following:

– Suppose that the supply s_{n+1} of the next year depends as a power function on the price p_n this year, and the higher the p_n, the greater the s_{n+1}: $s_{n+1} = ap_n^m - b$, where a and b are positive constants, and m belongs from 0 to 1, unchanged throughout the entire analyzed period of time.

– Suppose that next year's demand d_{n+1} depends exponentially on the price p_{n+1} in the same year, and the higher the price p_{n+1}, the lower the demand $d_{n+1}: d_{n+1} = g\exp(-cp_{n+1})$, where c and g are positive constants that remain unchanged throughout the entire analyzed time period.

Mathematical formulation of the problem. Assuming the value p_0 is given, find a sequence of values $p_1, p_2, p_3, \ldots$, satisfying the following system of equations:

$$s_{n+1} = ap_n^m - b, \tag{10}$$

$$d_{n+1} = g\exp(-cp_{n+1}), \tag{11}$$

$$s_{n+1} = d_{n+1}. \tag{12}$$

The solution of the problem. Substituting equations (10) and (11) into equation (12) we get:

$$ap_n^m - b = g\exp\left(-cp_{n+1}\right). \tag{13}$$

Transforming and introducing new constants: $A = a/g > 0, B = b/g > 0$ and $\mathrm{C} = \mathrm{c}^{-1} < 0$ we get:

$$p_{n+1} = C \cdot \ln\left(Ap_n^m - B\right). \tag{14}$$

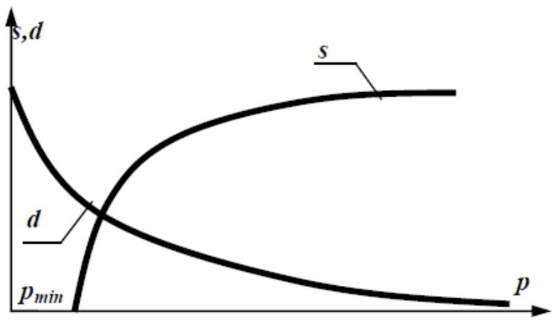

Figure 5. Minimum price and maximum possible demand

Source: compiled by the author

Since equation (14) does not have an analytical solution, we use direct modeling [3]. To do this, we will develop an algorithm that will allow us to determine the sequence of prices.

Conclusions. This paper examines the problem of constructing a mathematical model of supply and demand. Two models were investigated: linear and nonlinear. The linear model is convenient, but the market is much more complex, and you need to come up with a different model for it. It must be taken into account that supply cannot grow forever, since it is impossible to create an infinite amount of products. Supply and demand at some point react more sharply to price changes. The results of coefficient analysis for the nonlinear model were obtained. It is certain that the coefficient g must be greater than the coefficients a, b, c, with $A, B \approx 10^{-2}$, and $C \approx g$. It was also discovered that when coefficients a, b, g are greater than 10^{4}, the model does not work. It should be noted that the period for calculating prices should not be too long, since the coefficients may change over a long period of time. Thus, ready-to-use formulas for price growth and fall are obtained.

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