АВІАЦІЙНА ТА РАКЕТНО-КОСМІЧНА ТЕХНІКА

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OPTIMIZATION OF AIRCRAFT BEARING SURFACE BASED ON THE SOLUTION OF COUPLED EQUATIONS

The rational choice of bearing surfaces shapes is the main task when designing subsonic aircraft. The successful choice of the bearing surface shape to a great extend derivate the obtainment of high aerodynamic characteristics of the bearing surface and the vehicle as a whole.

Nowadays methods of aerodynamic design can be divided into experimental and numerical. Experimental methods are based on the results of numerous experiments and the obtained experience of the researcher. This approach is a high cost one, time consuming and does not guarantee an ultimate towards the solution of aerodynamic design problems. Numerous methods are based on the use of the mathematical tool of fluid mechanics and make it possible to define the optimal shape for a given flow state. These methods are at a low cost, faster and permit to find the optimal solution.

The research investigates the task of optimizing bearing surfaces for a stationary incompressible viscous fluid flow, which is characterized by average according to Reynolds – Navier-Stokes equations.

Nowadays numerical methods of aerodynamic design can be divided into two groups: inverse methods and optimization methods.

Inverse methods make it possible to define the aerodynamic shape for a given pressure or velocity distribution.

Unlike inverse methods, numerous optimization methods do not require a specific pressure or velocity field and can be formulated for a wide range of aerodynamic design problems. They can be divided into two groups: without gradient and gradient methods.

The most effective in the group of gradient methods is the method based on the solution of coupled equations. It allows to calculate the gradient by means of singelfold solved direct problems and coupled equations. Meanwhile, the time spent on calculating the gradient does not independent of the number of design variables.

Key words: *method, solution of coupled equations, reference, gradient, optimization, bearing surface.*

Statement of the task. Functional limitations *R(w,s)* is the Navier-Stokes equation for a two-dimensional stationary incompressible viscous flow. Let's mark cartesian reference as x_1, x_2, and the components of the velocity vector $-u_1$, u_2, we will also consider a summation for indices which are repeated *i(i=1,2)*. Then the Navier-Stokes equations can be represented as follows:

$$\frac{\partial f_i}{\partial x_i} - \frac{\partial f_i^v}{\partial x_i} = 0 \tag{1.1}$$

where

$$f_i = \begin{Bmatrix} u_i \\ pu_i u_1 + p\delta_{i1} \\ pu_i u_2 + p\delta_{i2} \end{Bmatrix}, \qquad f_i^v = \begin{Bmatrix} 0 \\ \sigma_{ij}\delta_{j1} \\ \sigma_{ij}\delta_{j2} \end{Bmatrix}, \tag{1.2}$$

$$\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right); \tag{1.3}$$

p – pressure; ρ – density; μ – dynamic viscosity.

A vector is taken as the variable flow field *w(s)*:

$$w = \begin{Bmatrix} p \\ u_1 \\ u_2 \end{Bmatrix}. \tag{1.4}$$

For further derivation of the coupled equations, the vector equation (1.1) must be represented in the computational space with the reference ξ_1, ξ_2 such that the contour of the body which is investigated lies on the axis ξ_1 (Fig. 1).

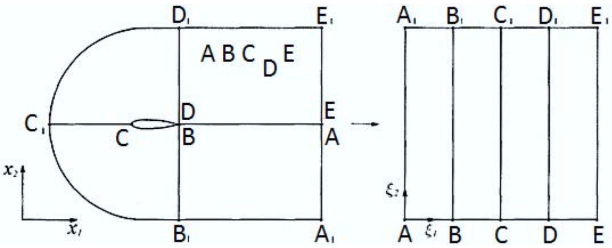

Fig. 1. Physical and computing areas

In order to represent the Navier-Stokes equations in the computational coordinate system, we have the following correlations:

$$x_i = x_i(\xi_1, \xi_2);\xi_i = \xi_i(x_1, x_2);$$

$$\begin{cases} \mathrm{d}x_1 = \dfrac{\partial x_1}{\partial \xi_1}\mathrm{d}\xi_1 + \dfrac{\partial x_1}{\partial \xi_2}\mathrm{d}\xi_{2,} \\ \mathrm{d}x_2 = \dfrac{\partial x_2}{\partial \xi_1}\mathrm{d}\xi_1 + \dfrac{\partial x_2}{\partial \xi_2}\mathrm{d}\xi_{2,} \end{cases} \tag{1.5}$$

$$\begin{cases} \mathrm{d}\xi_1 = \dfrac{\partial \xi_1}{\partial x_1}\mathrm{d}x_1 + \dfrac{\partial \xi_1}{\partial x_2}\mathrm{d}x_2, \\ \mathrm{d}\xi_2 = \dfrac{\partial \xi_2}{\partial x_1}\mathrm{d}x_1 + \dfrac{\partial \xi_2}{\partial x_2}\mathrm{d}x_2 \end{cases}. \tag{1.6}$$

Let's solve the system (2.5) regarding $\mathrm{d}\xi_1$, $\mathrm{d}\xi_2$, then we will obtain

$$\begin{cases} \mathrm{d}\xi_1 J = \dfrac{\partial x_2}{\partial \xi_2}\mathrm{d}x_1 + \dfrac{\partial x_1}{\partial \xi_2}\mathrm{d}x_{2,} \\ \mathrm{d}\xi_2 J = -\dfrac{\partial x_2}{\partial \xi_1}\mathrm{d}x_1 + \dfrac{\partial x_1}{\partial \xi_1}\mathrm{d}x_{2,} \end{cases}. \tag{1.7}$$

where J – transition is determined according to formula

$$J = \begin{vmatrix} \dfrac{\partial x_1}{\partial \xi_1} & \dfrac{\partial x_1}{\partial \xi_2} \\ \dfrac{\partial x_2}{\partial \xi_1} & \dfrac{\partial x_2}{\partial \xi_2} \end{vmatrix} = \frac{\partial x_1}{\partial \xi_1}\frac{\partial x_2}{\partial \xi_2} - \frac{\partial x_1}{\partial \xi_2}\frac{\partial x_2}{\partial \xi_1}. \tag{1.8}$$

On the basis of correlations (1.6) and (1.7) we obtain

$$\frac{\partial \xi_1}{\partial x_1}J = \frac{\partial x_2}{\partial \xi_2}; \qquad \frac{\partial \xi_1}{\partial x_2}J = -\frac{\partial x_1}{\partial \xi_2};$$

$$\frac{\partial \xi_2}{\partial x_1}J = \frac{\partial x_2}{\partial \xi_1}; \qquad \frac{\partial \xi_2}{\partial x_2}J = \frac{\partial x_1}{\partial \xi_1}. \tag{1.9}$$

Equation (1.1) can be putted down in the computational reference as follows

$$\frac{\partial f_i}{\partial \xi_j}\frac{\partial \xi_j}{\partial x_i} - \frac{\partial f_i^v}{\partial \xi_j}\frac{\partial \xi_j}{\partial x_i} = 0. \tag{1.10}$$

Multiply equation (1.10) by J and use correlation (1.9), then we will obtain

$$\frac{\partial f_1}{\partial \xi_1}\frac{\partial x_2}{\partial \xi_2} - \frac{\partial f_1}{\partial \xi_2}\frac{\partial x_2}{\partial \xi_1} - \frac{\partial f_2}{\partial \xi_1}\frac{\partial x_1}{\partial \xi_2} + \frac{\partial f_2}{\partial \xi_2}\frac{\partial x_1}{\partial \xi_1} -$$

$$-\left(\frac{\partial f_1^v}{\partial \xi_1}\frac{\partial x_2}{\partial \xi_2} - \frac{\partial f_1^v}{\partial \xi_2}\frac{\partial x_2}{\partial \xi_1} - \frac{\partial f_2^v}{\partial \xi_1}\frac{\partial x_1}{\partial \xi_2} + \frac{\partial f_2^v}{\partial \xi_2}\frac{\partial x_1}{\partial \xi_1}\right) = 0,$$

or

$$\frac{\partial F_i}{\partial \xi_i} - i\frac{\partial F_i^v}{\partial \xi_i} = 0, \tag{1.11}$$

$F_i = S_{ij}f_j$ – convective flow;
$F^v_i = S_{ij}f^v_j$ – diffusion flow;

$$F_i = \begin{Bmatrix} S_{ij}u_j \\ S_{ij}\rho u_j u_1 + S_{i1}\rho \\ S_{ij}\rho u_j u_2 + S_{i2}\rho \end{Bmatrix}, \quad F_i^v = \begin{Bmatrix} 0 \\ S_{ij}\sigma_{j1} \\ S_{ij}\sigma_{j2} \end{Bmatrix}, \tag{1.12}$$

$$S = \begin{pmatrix} \dfrac{\partial x_2}{\partial \xi_2} & -\dfrac{\partial x_1}{\partial \xi_2} \\ -\dfrac{\partial x_2}{\partial \xi_1} & \dfrac{\partial x_1}{\partial \xi_1} \end{pmatrix}. \tag{1.13}$$

Thus, the constraint functions can be written as

$$R = \frac{\partial F_i}{\partial \xi_i} - \frac{\partial F_i^v}{\partial \xi_i} = 0. \tag{1.14}$$

We will obtain a variation of the constraint functions $\delta R(w,s)$. According to the fact that in the computational space the shape of the body and, accordingly, the area D_ξ remain unchanged with variations of the shape in physical space, then for any point of the computational area D_ξ can be put as follows:

$$\delta R = R(w + \delta w, s + \delta s) - R(w, s) =$$

$$= \frac{\partial F_i(w + \delta w, s + \delta s)}{\partial \xi_i} - \frac{\partial F_i^v(w + \delta w, s + \delta s)}{\partial \xi_i} - \left(\frac{\partial F_i(w,s)}{\partial \xi_i} - \frac{\partial F_i^v(w,s)}{\partial \xi_i}\right)$$

$$= \frac{\partial}{\partial \xi_i}\left(F_i(w + \delta w, s + \delta s) - F_i(w,s)\right) - \frac{\partial}{\partial \xi_i}\left(F_i^v(w + \delta w, s + \delta s) - F_i^v(w,s)\right) = 0,$$

or

$$\delta R = \frac{\partial(\delta F_i)}{\partial \xi_i} - \frac{\partial(\delta F_i^v)}{\partial \xi_i} = 0. \tag{1.15}$$

Let's put down the variations of flows as follows

$$\delta F_i = \delta F_{iI} + \delta F_{iII}, \delta F_i^v = \delta F_{iI}^v + \delta F_{iII}^v, \tag{1.16}$$

where the variations with the index I are the contributions related to the change of the variables of the flow field δw, and with an index II – contributions related to the change in body shape δs.

Let's consider objective functional I, which can be represented as an integral outside the margin $B_{\xi w}$

$$I = \int_{B_{\xi w}} M(w,s)\,\mathrm{d}B_\xi, \tag{1.17}$$

where $B_{\xi w}$ – body contour in computing space;

$M(w,s)$ – subintegral function of cost functional. The type of the subintegral function depends on the particular formulation of the optimization task. The expression for the variation of the cost functional can be represented as follows:

$$\delta I = \int_{B_{\xi w}} \delta M \mathrm{d}B_\xi. \tag{1.18}$$

We multiply equation (1.15) by the vector of Lagrange multipliers at each point of the area and integrate along it, as a result we obtain:

$$\int_{D_\xi} \Psi^T \frac{\partial}{\partial \xi_i}\left(\delta F_i - \delta F_i^v\right) \mathrm{d}D_\xi = 0\,, \qquad (1.19)$$

where

$$\mathrm{d}D_\xi = d\xi_1 d\xi_2$$

Let's assume that the function ψ is continuous and differentiable, then integrating the expression (1.19) by parts and applying the Gauss theorem, we have

$$\int_{D_\xi} \Psi^T \frac{\partial}{\partial \xi_i}\left(\delta F_i - \delta F_i^v\right) \mathrm{d}D_\xi =$$

$$= \int_{B_\xi} n_i^\xi \Psi^T \left(\delta F_i - \delta F_i^v\right) \mathrm{d}B_\xi - \int_{D_\xi} \frac{\partial \Psi^T}{d\xi_i}\left(\delta F_i - \delta F_i^v\right) \mathrm{d}D_\xi = 0\,, \quad (1.20)$$

Since the expression (1.20) is equals zero, it can be subtracted from the expression for the variation of the cost functional (1.18) and obtain

$$\delta \mathrm{I} = \int_{B_{\xi w}} \delta M \mathrm{d}B_\xi - \int_{B_\xi} n_i^\xi \Psi^T \left(\delta F_i - \delta F_i^v\right) \mathrm{d}B_\xi + \int_{D_\xi} \frac{\partial \Psi^T}{d\xi_i}\left(\delta F_i - \delta F_i^v\right) \mathrm{d}D_\xi\,.$$

Based on the assumption that at the outer boundary of the area of flows variation which is considered, as a result of the change in the shape of the body equal zero, can be shown as follows:

$$\delta I = \int_{B_{\xi w}} \left[\delta M - n_i^\xi \Psi^T \left(\delta F_i - \delta F_i^v\right)\right] \mathrm{d}B_\xi + \int_{D_\xi} \frac{\partial \Psi^T}{d\xi_i}\left(\delta F_i - \delta F_i^v\right) \mathrm{d}D_\xi\,.$$

Going forward, the following representation of the variation of the cost functional will be more convenient

$$\delta I = \int_{B_{\xi w}} \left[\delta M - n_i^\xi \Psi^T \left(\delta F_i - \delta F_i^v\right)\right] \mathrm{d}B_\xi + \int_{D_\xi} \frac{\partial \Psi^T}{\partial \xi_i}\left(\delta F_{iI} - \delta F_{iI}^v\right) \mathrm{d}D_\xi +$$

$$\int_{D_\xi} \frac{\partial \Psi^T}{d\xi_i}\left(\delta F_{iII} - \delta F_{iII}^v\right) \mathrm{d}D_\xi\,. \qquad (1.21)$$

It is necessary to find such functions *ψ*, that the first two integrals of expression (1.21) became zero. Let's transform the second integral of expression (1.21)

$$\int_{D_\xi} \frac{\partial \Psi^T}{d\xi_i}\left(\delta F_{iI} - \delta F_{iI}^v\right) \mathrm{d}D_\xi = \int_{D_\xi} \frac{\partial \Psi^T}{\partial \xi_i} \delta F_{iI} \mathrm{d}D_\xi - \int_{D_\xi} \frac{\partial \Psi^T}{\partial \xi_i} \delta F_{iI}^v \mathrm{d}D_\xi\,. \qquad (1.22)$$

Let's examine the first integral on the right side of the expression (1.22). According to the correlation (1.12), we state

$$\delta F_i = \delta\left(S_{iJ} f_j\right) = \delta S_{ij} f_j + S_{ij} \delta f_j\,,$$

because at each point in the computational space

$$D_\xi f_j = f_j(w)\,,$$

then

$\delta f_j = \frac{\partial f_j}{\partial w}\delta w$, that's why $F_{iI} = S_{iJ} \frac{\partial f_j}{\partial w}\delta w$; $\delta F_{iII} = \delta S_{ij} f_j$, (1.23)

where

$$\delta w = \begin{pmatrix} \delta p \\ \delta u_1 \\ \delta u_2 \end{pmatrix}. \qquad (1.24)$$

Let's define the components of vector δF_{iI}. According to the correlation (1.12), we state

$$\delta F_i = \begin{Bmatrix} S_{i1}\delta u_1 + S_{i2}\delta u_2 \\ S_{i1}\left(2\rho u_1 \delta u_1 + \delta\rho\right) + S_{i2}\left(\rho u_2 \delta u_1 + \rho u_1 \delta u_2\right) \\ S_{i1}\left(\rho u_2 \delta u_1 + \rho u_1 \delta u_2\right) + S_{i2}\left(2\rho u_2 \delta u_2 + \delta\rho\right) \end{Bmatrix}, \qquad (1.25)$$

then the first integral on the right side of the expression (2.22) can be represented as follows

$$\int_{D_\xi} \frac{\partial \Psi^T}{\partial \xi_i} \delta F_{iI} \mathrm{d}D_\xi = \int_{D_\xi} \left(\frac{\partial \Psi_2}{\partial \xi_i} S_{i1} + \frac{\partial \Psi_3}{\partial \xi_i} S_{i2}\right) \delta\rho \mathrm{d}D_\xi +$$

$$+ \int_{D_\xi} \left[\frac{\partial \Psi_1}{\partial \xi_i} S_{i1} + \frac{\partial \Psi_2}{\partial \xi_i}(S_{i1} 2\rho u_1 + S_{i2}\rho u_2) + \frac{\partial \Psi_3}{\partial \xi_i} S_{i1}\rho u_2\right] = \delta u_1 \mathrm{d}D_\xi +$$

$$+ \int_{D_\xi} \left[\frac{\partial \Psi_1}{\partial \xi_i} S_{i2} + \frac{\partial \Psi_2}{\partial \xi_i} S_{i2}\rho u_1 + \frac{\partial \Psi_3}{\partial \xi_i}(S_{i1}\rho u_1 + S_{i2} 2\rho u_2)\right] \delta u_2 \mathrm{d}D_\xi\,. \quad (1.26)$$

Let's consider the second integral on the right side of the expression (1.22)

$$-\int_{D_\xi} \frac{\partial \Psi^T}{\partial \xi_i} \delta F_{iI}^v \mathrm{d}D_\xi\,.$$

Let's determine the contribution from the viscous terms of the equations of motion to the coupled equations

$$-\int_{D_\xi} \frac{\partial \Phi_k}{\partial_{\xi_i}} \left(\delta S_{ij}\sigma_{kj} + S_{ij}\delta\sigma_{kj}\right) \mathrm{d}D_\xi\,,$$

where

$$\sigma_{kj} = \mu\left(\frac{\partial u_k}{\partial x_j} + \frac{\partial u_j}{\partial x_k}\right).$$

The velocity derivatives can be represented as

$$\frac{\partial u_i}{\partial x_j} = \frac{\partial u_i}{\partial \xi_l}\frac{\partial \xi_l}{\partial x_j} = \frac{S_{lj}}{J}\frac{\partial u_i}{\partial \xi_l}\,.$$

Then for variation of stress, we obtain

$$\delta\sigma_{kj} = \mu\left(\frac{S_{lj}}{J}\frac{\partial}{\partial \xi_l}\delta u_k + \frac{S_{lk}}{J}\frac{\partial}{\partial \xi_l}\delta u_j\right) + \mu\left(\delta\left(\frac{S_{lj}}{J}\right)\frac{\partial u_k}{\partial \xi_l} + \delta\left(\frac{S_{lk}}{J}\right)\frac{\partial u_j}{\partial \xi_l}\right),$$

or

$$\delta\sigma_{kj} = \delta\sigma_{kj}^I + \delta\sigma_{kj}^{II}\,, \qquad (1.27)$$

where

$$\delta\sigma_{kj}^I = \mu\left(\frac{S_{lj}}{J}\frac{\partial}{\partial \xi_l}\delta u_k + \frac{S_{lk}}{J}\frac{\partial}{\partial \xi_l}\delta u_j\right);$$

$$\delta\sigma_{kj}^{II} = \mu\left(\delta\left(\frac{S_{lj}}{J}\right)\frac{\partial u_k}{\partial \xi_l} + \delta\left(\frac{S_{lk}}{J}\right)\frac{\partial u_j}{\partial \xi_l}\right). \qquad (1.28)$$

Thus, it can be represented as follows:

$$\delta F_{iI}^{v1} = 0\,; \qquad \delta F_{iII}^{v1} = 0\,;$$

$$\delta F_{iI}^{v2} = S_{ij}\delta\sigma_{1j}^I\,; \qquad \delta F_{iII}^{v2} = \delta S_{ij}\sigma_{1j} + S_{ij}\delta\sigma_{1j}^{II}\,;$$

$$\delta F_{iI}^{v3} = S_{ij}\delta\sigma_{2j}^I\,; \qquad \delta F_{iII}^{v3} = \delta S_{ij}\sigma_{2j} + S_{ij}\delta\sigma_{2j}^{II}\,; \quad (1.29)$$

Taking into consideration, the obtained correlations, the contribution from the viscous terms of the equations of motion to the coupled equations is represented as

$$-\int_{D_\xi} \frac{\partial \Psi^T}{\partial \xi_i} \delta F_{iI}^v \mathrm{d}D_\xi = -\int_{D_\xi} \frac{\partial \Phi_k}{\partial_{\xi_i}} S_{ij}\mu\left(\frac{S_{lj}}{J}\frac{\partial}{\partial \xi_l}\delta u_k + \frac{S_{lk}}{J}\frac{\partial}{\partial \xi_l}\delta u_j\right) \mathrm{d}D_\xi\,. \qquad (1.30)$$

Integrating the expression (1.30) by parts and using the Ostrogradsky-Gauss theorem, we obtain

$$-\int_{D_\xi} \frac{\partial \Phi_k}{\partial_{\xi_i}} S_{ij}\mu\left(\frac{S_{lj}}{J}\frac{\partial}{\partial \xi_l}\delta u_k + \frac{S_{lk}}{J}\frac{\partial}{\partial \xi_l}\delta u_j\right)\mathrm{d}D_\xi =$$

$$= -\int_{B_\xi} \frac{\partial \Phi_k}{\partial_{\xi_i}} n_l^\xi S_{ij}\mu\left(\frac{S_{lj}}{J}\delta u_k + \frac{S_{lk}}{J}\delta u_j\right)\mathrm{d}B_\xi +$$

$$+\int_{D_\xi}\delta u_k \frac{\partial}{\partial \xi_l}\left[S_{ij}S_{lj}\frac{\mu}{J}\frac{\partial \Phi_k}{\partial_{\xi_i}}\right]\mathrm{d}D_\xi + \int_{D_\xi}\delta u_j \frac{\partial}{\partial \xi_l}\left[S_{ij}S_{lk}\frac{\mu}{J}\frac{\partial \Phi_k}{\partial_{\xi_i}}\right]\mathrm{d}D_\xi.$$

We consider that the integral along the boundary becomes zero, since the variations δuk=0 on the boundary. Also, replacing the corresponding indices in the last two integrals, we have

$$-\int_{D_\xi}\frac{\partial \Psi^T}{\partial \xi_i}\delta F_{il}^v \mathrm{d}D_\xi = \int_{D_\xi}\delta u_k \frac{\partial}{\partial \xi_l}\left[S_{lj}\mu\left(\frac{S_{ij}}{J}\frac{\partial \Phi_k}{\partial_{\xi_i}} + \frac{S_{ik}}{J}\frac{\partial \Phi_j}{\partial_{\xi_i}}\right)\right]\mathrm{d}D_\xi$$

We will note that

$$\frac{\partial S_{lj}}{\partial \xi_l} = 0$$

$$\frac{S_{ij}}{J}\frac{\partial \Phi_k}{\partial \xi_i} = \frac{\partial \Phi_k}{\partial x_j},$$

then we will get

$$-\int_{D_\xi}\frac{\partial \Psi^T}{\partial \xi_i}\delta F_{il}^v \mathrm{d}D_\xi = \int_{D_\xi}\delta u_k\left[S_{lj}\frac{\partial}{\partial \xi_l}\left(\mu\left(\frac{\partial \Phi_k}{\partial x_j} + \frac{\partial \Phi_j}{\partial x_k}\right)\right)\right]\mathrm{d}D_\xi .$$

The obtained coupled equations do not depend on the form of the objective functional and can be used to solve volitional two-dimensional problems of optimizing aerodynamic shapes for an uncompressed stationary viscous fluid flow.

Main material presenting

To solve the coupled equations, it is necessary to set boundary conditions which can be obtained from the first integral of the expression (1.21)

$$\int_{B_{\xi w}}\left[\delta M - n_i^\xi \Psi^T\left(\delta F_i - \delta F_i^v\right)\right]\mathrm{d}B_\xi. \qquad (1.31)$$

As an example, let's consider three different cost functionals and will obtain from expression (2.31) the corresponding boundary conditions for the coupled equations.

Let's assume that the cost functional will be the resultant force vector acting on the bearing surface, projected in a certain direction

$$I = X_a q_1 + Y_a q_2 = q_1\int_{B_w} n_j(\delta_{j1}p - \sigma_{j1})dB + q_2\int_{B_w} n_j(\delta_{j2}p - \sigma_{j2})dB, \qquad (1.32)$$

where

B_w – contour of the body in physical space;

n_j – vector of the normal towards the element dB in physical space.

Let's imagine the expression (1.32) as an integral along the boundary in the computational space. The normal vector n towards the contour of the body can be represented as

$$\bar{n} = \frac{\frac{\partial \bar{r}}{\partial \xi_1}\times \bar{k}}{\left|\frac{\partial \bar{r}}{\partial \xi_1}\times \bar{k}\right|} = \frac{\bar{\mathrm{i}}\frac{\partial x_2}{\partial \xi_1} - \bar{j}\frac{\partial x_1}{\partial \xi_1}}{\sqrt{\left(\frac{\partial x_2}{\partial \xi_1}\right)^2 + \left(\frac{\partial x_1}{\partial \xi_1}\right)^2}} = -\frac{\bar{\mathrm{i}}S_{21} + \bar{j}S_{22}}{|S_2|}, \text{ тобто}$$

$$n_j = -\frac{S_{2j}}{|S_2|} \text{ ,де } |s_2| = \sqrt{s_{2j}s_{2j}}.$$

The element dB can be represented as

$$\mathrm{d}B = \left|\frac{\partial \bar{r}}{\partial \xi_1}d\xi_1 \times \bar{k}\right| = \left|\frac{\partial \bar{r}}{\partial \xi_1}\times \bar{k}\right| d\xi_1 = |s_2|\, d\xi_1 .$$

Thus, the cost functional can be written as

$$I = q_1\int_{B_{\xi w}} S_{2j}(\delta_{j1}p - \sigma_{j1})d\xi_1 - q_2\int_{B_{\xi w}} S_{2j}(\delta_{j2}p - \sigma_{j2})d\xi_1. \quad (1.33)$$

According to the expression (1.17)

$$M(w,s) = -q_1 S_{2j}\left(\delta_{j1}p - \sigma_{j1}\right) - q_2 S_{2j}\left(\delta_{j2}p - \sigma_{j2}\right); dB_\xi = d\xi_1. \quad (1.34)$$

Expression (1.31) can be represented in the form

$$\int_{B_{\xi w}}\left[\delta M - n_i^\xi \psi^T\left(\delta F_i - F_i^v\right)\right]dB_\xi = \int_{B_{\xi w}}\left[\delta M - \psi^T\delta\left(F_2 - F_2^v\right)\right]d\xi_1 =$$

$$= -\int_{B_{\xi w}} q_1\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}} q_2\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1 +$$

$$+\int_{B_{\xi w}}\psi_1\delta\left(s_{2j}pu_j\right)d\xi_1 + \int_{B_{\xi w}}\psi_2\delta\left(S_{2j}pu_ju_1 + S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 +$$

$$+\int_{B_{\xi w}}\psi_3\delta\left(S_{2j}pu_ju_2 + S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1.$$

Let's consider the boundary conditions for the fluid motion equations u_i=0, then we will obtain

$$\int_{B_{\xi w}}\left[\delta M + \psi^T\delta\left(F_2 - F_2^v\right)\right]d\xi_1 =$$

$$-\int_{B_{\xi w}} q_1\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}} q_2\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1 + \quad (1.35)$$

$$+\int_{B_{\xi w}}\psi_2\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 + \int_{B_{\xi w}}\psi_3\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1.$$

Upon solving the system of coupled differential equations with the obtained boundary conditions, the variation of the cost functional can be calculated as

$$\delta I = \int_{D_\xi}\frac{\partial \psi^T}{\partial \xi_i}\left(\delta F_{iII} - \delta F_{iII}^v\right)\mathrm{d}D_\xi. \qquad (1.36)$$

Variation of flows can be represented as below

$$\delta F_{iII} = \frac{\partial F_i}{\partial s}\delta s\,;\; \delta F_{iII}^v = \frac{\partial F_i^v}{\partial s}\delta s.$$

Then the expression (1.36) can be given the form

$$\delta I = \int_{D_\xi}\frac{\partial \psi^T}{\partial \xi_i}\left(\frac{\partial F_i}{\partial s} - \frac{\partial F_i^v}{\partial s}\right)\delta s \mathrm{d}D_\xi$$

Let's consider how the next task of optimization maximizes the aerodynamic quality of the bearing surface, i.e. the cost functionality

$$I = -\frac{Y_a}{X_a}.$$

Let's write down a variation of this functional:

$$\delta I = \delta\left(-\frac{Y_a}{X_a}\right) = -\frac{1}{X_a}\delta Y_a + \frac{Y_a}{X_a^2}\delta X_a =$$

$$= -\frac{Y_a}{X_a^2}\int_{B_{\xi w}} \delta\left(s_{21}p - s_{2j}\sigma_{j1}\right)\mathrm{d}\xi_1 + \frac{1}{X_a}\int_{B_{\xi w}} \delta\left(s_{22}p - s_{2j}\sigma_{j2}\right)\mathrm{d}\xi_1.$$

According to expression (1.31), we obtain

$$\int_{B_{\xi w}} \left[\delta M - n_i^{\xi}\psi^T\left(\delta F_i - \delta F_i^v\right)\right]\mathrm{d}B_{\xi} =$$

$$= -\int_{B_{\xi w}} \frac{Y_a}{X_a^2}\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)\mathrm{d}\xi_1 + \int_{B_{\xi w}} \frac{1}{X_a}\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)\mathrm{d}\xi_1 +$$

$$+ \int_{B_{\xi w}} \psi_2\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)\mathrm{d}\xi_1 + \int_{B_{\xi w}} \psi_{w3}\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)\mathrm{d}\xi_1.$$

From this equation can be seen, that if we accept

$$\psi_2 = \frac{Y_a}{X_a^2}, \psi_3 = -\frac{1}{X_a}$$

on the boundary of the object, then the integral which is considered will turn to zero. As in the previous problem, the multiplier ψ_1 on the boundary of the object and multipliers $\psi_1\psi_2$ і ψ_3 on the outer boundary can be chosen randomly. Having solved the coupled equations with the obtained boundary conditions, the variation of the cost functional can be calculated by formula (1.36).

As an example, let's consider another definition of the profile optimization problem. It is necessary to minimize the module of the aerodynamic moment $|M_z|$, current on the profile with respect to the point with coordinates (x_0,y_0). The corresponding cost functional can be represented as follows

$$I = \frac{1}{2}M_z^2 = \frac{1}{2}\left[\int_{B_w}(x - x_0)n_j\left(\delta_{j2}\sigma_{j2}\right)dB - \int_{B_w}(y - y_0)n_j\left(\delta_{j1}p - \sigma_{j1}\right)dB\right]^2 =$$

$$\frac{1}{2}\left[\int_{B_{\xi w}}(y - y_0)S_{2j}\left(\delta_{j1}p - \sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}}(x - x_0)S_{2j}\left(\delta_{j2}p - \sigma_{j2}\right)d\xi_1\right]^2.$$

Let's write the variation of the cost functional as

$$\delta I = M_z\left[\int_{B_{\xi w}}(y - y_0)\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}}(x - x_0)\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1\right] +$$

$$+M_z\left[\int_{B_{\xi w}}\delta y\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}}\delta x\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1\right].$$

From expression (1.31) we obtain

$$\int_{B_{\xi w}} \left[\delta M - n_i^{\xi}\psi^T\left(\delta F_i - \delta F_i^v\right)\right]\mathrm{d}B_{\xi} =$$

$$= \int_{B_{\xi w}} M_z(y - y_0)\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}} M_z(x - x_0)\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1 +$$

$$+ \int_{B_{\xi w}} \psi_2\delta\left(S_{21}p - S_{2j}\sigma_{j1}\right)\mathrm{d}\xi_1 + \int_{B_{\xi w}} \psi_3\delta\left(S_{22}p - S_{2j}\sigma_{j2}\right)\mathrm{d}\xi_1 +$$

$$+M_z\left[\int_{B_{\xi w}}\delta y\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}}\delta x\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1\right].$$

The same as in the previous task, multiplier ψ_1 on the boundary of the object and multipliers ψ_1 ψ_2 and ψ_3 on the outer boundary can be chosen randomly. Having solved the coupled equations with the obtained boundary conditions, the variation of the cost functional can be calculated via formula

$$\delta I = \int_{D_{\xi}} \frac{\partial\psi^T}{\partial\xi_i}\left(\partial F_{iII} - \partial F_{iII}^v\right)\mathrm{d}D_{\xi} +$$

$$+M_z\left[\int_{B_{\xi w}}\delta y\left(S_{21}p - S_{2j}\sigma_{j1}\right)d\xi_1 - \int_{B_{\xi w}}\delta x\left(S_{22}p - S_{2j}\sigma_{j2}\right)d\xi_1\right].$$

Conclusions. Numerous optimization methods, together with methods of computational hydro aerodynamics, make it possible to find such an aerodynamic shape that delivers a minimum of the cost functionality under given functional limitations. Cost functionality can be drag, aerodynamic quality, pressure ratio. Different types of aerodynamic or geometrical limitations can act as functional limitations, such as given lifting force, volume or area of the object that is needed to be optimized.

The boundary conditions of the coupled equations can be obtained for many other tasks of bearing surfaces optimization.

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Рагулін С.В., Шарабайко О.М., Лозовський В.Г. ОПТИМІЗАЦІЯ ФОРМ АЕРОДИНАМІЧНИХ ПРОФІЛІВ ЛІТАЛЬНИХ АПАРАТІВ НА ОСНОВІ РОЗВ'ЯЗАННЯ ПОВ'ЯЗАНИХ РІВНЯНЬ

При проектуванні дозвукових літаків важливим завданням є раціональний вибір форм несучих поверхонь. Вдалий вибір форми несучої поверхні, значною мірою обумовлює отримання високих аеродинамічних характеристик несучої поверхні і апарату в цілому.

Сучасні методи аеродинамічного проектування можна поділити на експериментальні та чисельні. Експериментальні методи спираються на результати численних експериментів та досвід дослідника. Цей підхід є дорогим, витратним за часом та не гарантує отримання оптимального розв'язання задач аеродинамічного проектування. Численні методи засновані на використанні математичного апарату механіки рідини та газу та дозволяють визначити оптимальну форму для заданого режиму течії. Ці методи є дешевшими, швидкими і дозволяють знаходити оптимальне рішення.

У роботі розглядається завдання оптимізації аеродинамічних профілів для стаціонарної нестискаємої в'язкої течії рідини, що описується середніми по Рейнольдсу рівняннями Навьє-Стокса.

Сучасні чисельні методи аеродинамічного проектування можна поділити на дві групи: зворотні методи та прямі методи оптимізації.

Зворотні методи дозволяють знайти аеродинамічну форму за заданим розподілом тиску або швидкості.

На відміну від зворотних методів, чисельні методи оптимізації не вимагають певного поля тиску або швидкості і можуть бути сформульовані для широкого класу аеродинамічних завдань проектування. Їх можна поділити на дві групи: безградієнтні та градієнтні методи.

Найбільш ефективним у групі градієнтних методів є метод на основі розв'язання пов'язаних рівнянь. Він дозволяє обчислити градієнт за допомогою одноразово вирішених прямих завдань і пов'язаних рівнянь. При цьому час, що витрачається на обчислення градієнта, практично не залежить від кількості проектних змінних.

Ключові слова: *метод, розв'язання пов'язаних рівнянь, система координат, градієнт, оптимізація, аеродинамічний профіль.*