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Investigation of Free Vibrations of Three-Layered Circular Shell, Supported by Annular Ribs of Rigidity

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Abstract: The construction of a mathematical model and the development of an algorithm of free vibrations investigation in the three-layered circular shell with a light-weight aggregate supported by annular rigidity ribs are considered in paper. The hypotheses of Kirchoff-Lyav are accepted for external bearing layers of shell and for aggregate there is accepted the linear law of tangential displacements change by thickness.

The boundary conditions of a shell region closed between the ribs are established. Using the boundary transition, conditions along the lines of the ribs, taking into account and without deformations of displacement in the ribs, but without taking into account the torsional rigidity in the ribs are determined. The equation of motion of supported three-layered shell is obtained. The frequencies of free vibrations were investigated and values of parameter of the first frequency of free vibrations for a shell, supported by one and three rigidity ribs, were calculated. There are given values depending on the physical and mechanical properties of materials and geometric dimensions of the shell, the curvature parameter, and the rigidity parameter of an aggregate.

Introduction.

Increasing the level of building industrialization requires the use of new efficient types of light economic construction structures. The three-layered shell based on hard-core styrofoam or oriented mineral wool and sheet materials is the best option of a durable and rigid construction. The three-layered shells with a lightweight aggregate are used in the construction, aviation and other industries widely. The three-layered shell consists of two external bearing relatively thin layers made of a durable material, between which a sufficiently thick layer of low-strength material with a small volume weight is placed. This layer is called an aggregate. Three-layered structures are distinguished by the type of filler, the material of bearing layers, the method of connection. Lightweight aggregates can be: oriented mineral wool, styrofoam, single and double corrugated tubes, tubular aggregate and so on. Such a widespread using of three-layered shells is explained by their high weight characteristics, which allow with the same weight to withstand significantly higher loads than can withstand single-layered.

Formulation of the Problem.

For such structures, the solution to the problem of free vibrations is essential. Information about the first frequency of free vibrations is necessary for solving many problems of dynamics. Significant numbers of works are devoted by calculations on free vibrations of three-layered unsupported shells [1-7].

The questions of vibrations of three-layered shell construction, supported by ribs of rigidity [8, 9, 10], are highlighted insufficiently.

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The aim of the work is to build a mathematical model and development an algorithm for studying free vibrations of a three-layered circular shell with a light aggregate, which is supported by annular rigidity ribs with hinge supporting of edges.

Materials and Methods.

Using the variational principle of Ostrogradsky-Hamilton, the variational equation of transverse vibrations of a three-layered shell of a symmetrical structure was obtained. The three-layered shell, supported by ribs in two mutually perpendicular directions, taking into account action of longitudinal forces in the planes of outer layers and ribs, was considered. The hypotheses of Kirchoff-Lyav was adopted for external bearing layers, and for an aggregate and ribs the linear law of tangential displacement changes in thickness was taken and the bend of the ribs in the vertical plane was taken into account [11].

The boundary conditions of the region of a shell closed between the ribs are established. Using the boundary transition, conditions along lines of the ribs, taking into account and without account deformations of displacement in the ribs, but without taking into account torsional rigidity of the ribs are obtained.

Research Results.

Let's consider free vibrations of a three-layered circular shell with a light transversally isotropic aggregate, supported by ring ribs of rigidity. The distance between the ribs, as well as rigidities of the ribs, is considered to be the same. At the same time, the ribs are symmetrically relatively to median surface of a three-layered shell (Fig. 1).

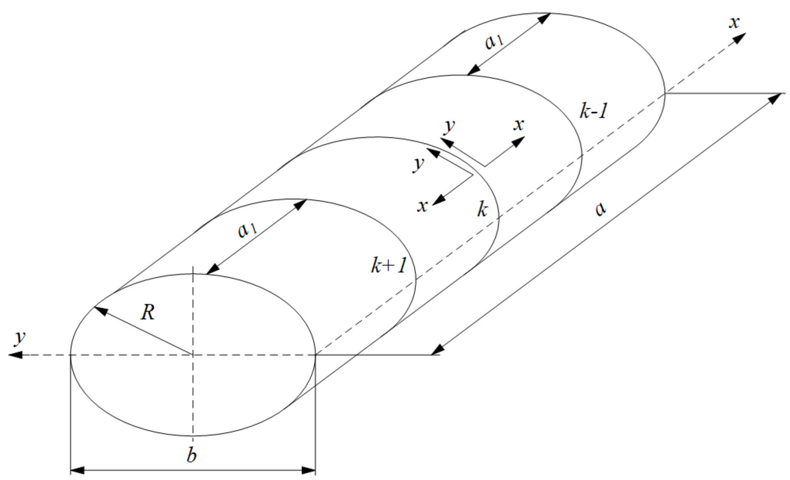

Fig. 1. Three-layered circular shell supported by annular ribs scheme

Differential equations of bending vibrations of a shell area, that is closed between the ribs, has a the form [11]

$$\nabla^4\Phi + \frac{\overline{B}}{R}\frac{\partial^2}{\partial x^2}\left(\phi - \frac{Bh}{G_3}\nabla^2\phi\right) = 0 \qquad (1)$$

$$\nabla^4\phi - \frac{1}{RD^*}\frac{\partial^2\Phi}{\partial x^2} - m_{\text{об}}\omega^2\left(\phi - \frac{Bh}{G_3}\nabla^2\phi\right) = 0 \qquad (2)$$

$$\Psi - \frac{1-\mu}{2G_3}Bh\nabla^2\Psi = 0 \qquad (3)$$

System of equations (1) and (2) is reduced to one solving equation (4) by introducing into consideration of function *F (x, y)* [12]:

$$\nabla^4\nabla^4 F+\frac{\overline{B}}{R^2D^*}\frac{\partial^4}{\partial x^4}\left(F-\frac{Bh}{G^3}\nabla^2 F\right)+\frac{\omega^2 m_{об}}{D^*}\left(\nabla^4 F-\frac{Bh}{G^3}\nabla^2\nabla^4 F\right)=0 \tag{4}$$

In equations (1) – (4) is denoted:

$B=\dfrac{E\delta}{\left(1-\mu^2\right)}$; $D^*=2BH^2$; $D=\dfrac{E\delta^3}{12\left(1-\mu^2\right)}$; $H=h+0,5\delta$; $\Phi=-\dfrac{\overline{B}}{R}\dfrac{\partial^2}{\partial x^2}\left(1-\dfrac{Bh}{G_3}\nabla^2\right)F$;

$\phi=\nabla^4 F$; $m_{об}=2\left(\rho_{н}\cdot\delta+\rho_{з.}\cdot h\right)$; $\delta, 2h-$ the thickness of an external layers and an aggregate.

Solution of solving equation (4) for area of shell closed between the ribs we will find for using a function:

$$F=f_1\left(x\right)\sin\frac{n}{R}y \tag{5}$$

Substituting indicated function (5) into solving equation (4), we obtain a differential equation which defines function $f_1(x)$:

$$\begin{aligned}&f_1^8\left(x\right)+k_0\left(m_\omega-\frac{\pi^2\alpha^2}{b^2}\right)f_1^6\left(x\right)+\frac{\pi^2\left(\pi^2\left(2+\alpha^2\right)-b^2m_\omega+k_0\left(\pi^2\alpha^2-b^2m_\omega\right)\right)}{b^4}f_1^4\left(x\right)+\\&+\frac{\pi^4k_0m_\omega}{b^4}f_1^2\left(x\right)+\frac{\pi^6\left(\pi^2-b^2\left(1+k_0\right)m_\omega\right)}{b^8}f_1\left(x\right)=0\end{aligned} \tag{6}$$

Solutions of a differential equation (6) we will find in the form:

$$f_1\left(x\right)=e^{\eta x} \tag{7}$$

Substituting a function (7) into differential equation (6) and shrinking on $e^{\eta x}$, we obtain characteristic equation which corresponds to differential equation (4):

$$\begin{aligned}&\left(1-k_0m_\omega\right)\beta^8+\left(m_\omega-k_0\alpha^2+3k_0m_\omega n^2-4n^2\right)\beta^6+\left(\alpha^2-2m_\omega n^2+k_0\alpha^2n^2-3k_0m_\omega n^4+6n^4\right)\beta^4+\\&+\left[m_\omega n^4\left(1+k_0n^2\right)-4n^6\right]+n^8=0\end{aligned} \tag{8}$$

Finally, a function F (x, y) is written as:

$$\begin{aligned}F=&\left\{\sin\left(\frac{\pi y}{b}\right)\left(\left(\cos\left(x\phi_1\right)\left(C_1+C_3\right)+\sin\left(x\phi_1\right)\left(C_2-C_4\right)\right)\rho_1^x+(\cos\left(x\phi_2\right)\left(C_5+C_7\right)+\right.\right.\\&\left.\left.+\sin\left(x\phi_2\right)\left(C_6-C_8\right)\right)\rho_2^x\right)\Big\}\end{aligned} \tag{9}$$

In equations (7 - 9) is denoted:

$\alpha^2=\dfrac{\overline{B}R^4}{R^2D^*}=\dfrac{\left(1-\mu^2\right)R^2}{H^2}$; $m_\omega=\dfrac{m_{об}\omega^2b^2}{D^*\pi^2\alpha_n^2}$; $k_0=\dfrac{Bh}{G_3R^2}$; $\beta=R\eta$;

$tg\phi_1 = \frac{r}{s}, \quad tg\phi_2 = \frac{d}{c}, \quad \rho_1 = \left|\sqrt{s^2 + r^2}\right|, \quad \rho_2 = \left|\sqrt{c^2 + d^2}\right|$ – here s, c – are valid , and r, d – are complex roots of the characteristic equation.

Solutions of equation (3) we will find in the form:

$$\Psi = f_2(x)\cos\frac{n}{R}y \tag{10}$$

Substituting a function (10) into equation (3), we obtain a differential equation for determining of the function $f_2(x)$. Solving this equation, we obtain:

$$f_2(x) = C_9\cos(\beta x) + C_{10}\sin(\beta x) \tag{11}$$

Here $\beta = \sqrt{\frac{2}{(1-\mu)k_0} + n^2}$

We write the boundary conditions for the case of free support of the shell, assuming that diaphragms are installed on edges of the shell:

$$(w)_{x=0,a} = \left(\frac{\partial u_\beta}{\partial x}\right)_{x=0,a} = (v_\beta)_{x=0,a} = (v_\alpha)_{x=0,a} = \left(\frac{\partial^2\Phi}{\partial y^2}\right)_{x=0,a} = 0 \tag{12}$$

We will accept for every area its coordinate axes [13]. The origin is located in the start of area and function $f_1(x)$ in the starts and in the ends of area (for $x = 0$ and $x=a_1$, where $a_1=a/m$, m – number of areas) is denoted as η_k and η_{k+1}, second derivative $f_1''(x)$ as μ_k and μ_{k+1}, fourth derivative $f_1^{IV}(x)$ as ζ_k and ζ_{k+1}, sixth derivative $f_1^{VI}(x)$ as ξ_k and ξ_{k+1}, second derivative $f_2''(x)$ as φ_k and φ_{k+1}.

In [14], the authors derived a system of equations for the determination of arbitrary constants Ci of solutions (9) and (10), which can be applied to this research:

$$\begin{aligned}
&\eta_k = C_1^k + C_3^k + C_5^k + C_7^k,\\
&\mu_k = (C_1^k + C_3^k)\left(p_5^2 - \phi_1^2\right) + (C_5^k + C_7^k)\left(p_6^2 - \phi_2^2\right) + 2\left(C_2^k - C_4^k\right)p_5\phi_1 + 2\left(C_6^k - C_8^k\right)p_6\phi_2,\\
&\zeta_k = (C_1^k + C_3^k)\left(p_5^4 - 6p_5^2\phi_1^2 + \phi_1^4\right) + (C_5^k + C_7^k)\left(p_6^4 - 6p_6^2\phi_2^2 + \phi_2^4\right) + \left(C_2^k - C_4^k\right)\times\\
&\times(4p_5^3\phi_1 - 4p_5\phi_1^3) + \left(C_6^k - C_8^k\right)(4p_6^3\phi_2 - 4p_6\phi_2^3),\\
&\xi_k = (C_1^k + C_3^k)\left(p_5^6 - 15p_5^4\phi_1^2 + 15p_5^2\phi_1^4 + \phi_1^6\right) + (C_5^k + C_7^k)\left(p_6^6 - 15p_6^4\phi_2^2 + 15p_6^2\phi_2^4 + \phi_2^4\right) +\\
&+\left(C_2^k - C_4^k\right)(6p_5^5\phi_1 - 20p_5^3\phi_1^3 + 6p_5\phi_1^5) + \left(C_6^k - C_8^k\right)(6p_6^5\phi_2 - 20p_6^3\phi_2^3 + 6p_6\phi_2^5),\\
&\phi_k = \beta^2 C_{10}^k,\\
&\eta_{k+1} = ((C_1^{k+1} + C_3^{k+1})p_1 + (C_2^{k+1} - C_4^{k+1})p_2)\rho_1^{a_1} + ((C_5^{k+1} + C_7^{k+1})p_3 + (C_6^{k+1} - C_8^{k+1})p_4)\rho_2^{a_1},
\end{aligned} \tag{13}$$

$$\mu_{k+1} = ((C_1^{k+1} + C_3^{k+1})\rho_1^{a_1}(p_1\left(p_5^2 - \phi_1^2\right) - 2p_2\phi_1 p_5) + (C_2^{k+1} - C_4^{k+1})\rho_1^{a_1}(p_2\left(p_5^2 - \phi_1^2\right) + 2p_1\phi_1 p_5) +$$
$$+((C_5^{k+1} + C_7^{k+1})\rho_2^{a_1}(p_3\left(p_6^2 - \phi_2^2\right) - 2p_6\phi_2 p_4) + (C_6^{k+1} - C_8^{k+1})p_4)\rho_2^{a_1}(p_4\left(p_6^2 - \phi_2^2\right) + 2p_6\phi_2 p_3), \tag{13}$$

$$\zeta_{k+1} = ((C_1^{k+1} + C_3^{k+1})\rho_1^{a_1}\left(p_1\left(p_5^4 - 6p_5^2\phi_1^2 + \phi_1^4\right) + 4p_2\left(\phi_1^3 p_5 - \phi_1 p_5^3\right)\right) + (C_2^{k+1} - C_4^{k+1})\times$$
$$\times\rho_1^{a_1}\left(p_2\left(p_5^4 - 6p_5^2\phi_1^2 + \phi_1^4\right) + 4p_1\left(\phi_1 p_5^3 - \phi_1^3 p_5\right)\right) + ((C_5^{k+1} + C_7^{k+1})\rho_2^{a_1}\left(p_3\left(p_6^4 - 6p_6^2\phi_2^2 + \phi_2^4\right) +\right.$$
$$\left.+4p_4\left(\phi_2^3 p_6 - \phi_2 p_6^3\right)\right) + (C_6^{k+1} - C_8^{k+1})\rho_2^{a_1}\left(p_4\left(p_6^4 - 6p_6^2\phi_2^2 + \phi_2^4\right) + 4p_3\left(\phi_2 p_6^3 - \phi_2^3 p_6\right)\right),$$

$$\xi_{k+1} = ((C_1^{k+1} + C_3^{k+1})\rho_1^{a_1}\left(p_1\left(p_5^6 - 15p_5^4\phi_1^2 + 15p_5^2\phi_1^4 - \phi_1^6\right) + p_2\left(20\phi_1^3 p_5^3 - 6\phi_1 p_5^5 - 6\phi_1^5 p_5\right)\right) +$$
$$+(C_2^{k+1} - C_4^{k+1})\rho_1^{a_1}\left(p_2\left(p_5^6 - 15p_5^4\phi_1^2 + 15p_5^2\phi_1^4 - \phi_1^6\right) + p_1\left(-20\phi_1^3 p_5^3 + 6\phi_1 p_5^5 + 6\phi_1^5 p_5\right)\right) +$$
$$+((C_5^{k+1} + C_7^{k+1})\rho_2^{a_1}\left(p_3\left(p_6^6 - 15p_6^4\phi_2^2 + 15p_6^2\phi_2^4 - \phi_2^6\right) + p_4\left(20\phi_2^3 p_6^3 - 6\phi_2 p_6^5 - 6\phi_2^5 p_6\right)\right) +$$
$$+(C_6^{k+1} - C_8^{k+1})\rho_2^{a_1}\left(p_4\left(p_6^6 - 15p_6^4\phi_2^2 + 15p_6^2\phi_2^4 - \phi_2^6\right) + p_3\left(-20\phi_2^3 p_6^3 + 6\phi_2 p_6^5 + 6\phi_2^5 p_6\right)\right),$$

$$\phi_{k+1} = \beta^2 C_{10}^{k+1}\sin\left(\beta a_1\right) + \phi_k\cos\left(\beta a_1\right).$$

In equation (13) is denoted:
$p_1 = \cos\left(\phi_1 b_1\right)$, $p_2 = \sin\left(\phi_1 b_1\right)$, $p_3 = \cos\left(\phi_2 b_1\right)$, $p_4 = \sin\left(\phi_2 b_1\right)$ $p_5 = \log\left(\rho_1\right)$, $p_6 = \log\left(\rho_2\right)$.

By revising of *(k-1)* area we accept the origin of coordinates at its end and direct the axis x to the opposite direction. Then for it $f_1(x)$ and $f_2(x)$ will have the same form as on the k area, and arbitrary constants (let's denote them as $C_i^{\kappa-1}$) will be determined from (11), if we will change in them $\eta_{k+1}, \mu_{k+1}, \zeta_{k+1}, \xi_{k+1}, \varphi_{k+1}$ by $\eta_{k-1}, \mu_{k-1}, \zeta_{k-1}, \xi_{k-1}, \varphi_{k-1}$.

Conditions on the lines of the annular ribs are written as:

$$\frac{\partial^4}{\partial y\partial x^3}\left(F - \frac{Bh}{G_3}\nabla^2 F\right)_{x=+0} + \frac{\partial^4}{\partial y\partial x^3}\left(F - \frac{Bh}{G_3}\nabla^2 F\right)_{x=-0} = \frac{\partial^5}{\partial y\partial x^4}\left(F - \frac{Bh}{G_3}\nabla^2 F\right)_{x=0};$$

$$\frac{\partial^4}{\partial y^2\partial x^2}\left(F - \frac{Bh}{G_3}\nabla^2 F\right)_{x=+0} = \frac{\partial^4}{\partial y^2\partial x^2}\left(F - \frac{Bh}{G_3}\nabla^2 F\right)_{x=-0}; \tag{14}$$

$$\left(\frac{\partial}{\partial y}\nabla^4 F + \frac{\partial\psi}{\partial x}\right)_{x=+0} = \left(\frac{\partial}{\partial y}\nabla^4 F + \frac{\partial\psi}{\partial x}\right)_{x=-0};$$

$$\left(\frac{\partial}{\partial x}\nabla^6 F\right)_{x=+0} + \left(\frac{\partial}{\partial x}\nabla^6 F\right)_{x=-0} = -\left\{\frac{D_{pk}}{D^*}\left(1 - \frac{Bh}{G_3}\nabla^2\right)\frac{\partial^4}{\partial x^4}\nabla^4 F + \frac{m_p\omega^2}{D^*}\left(1 - \frac{Bh}{G_3}\nabla^2\right)\frac{\partial^2}{\partial x^2}\nabla^4 F\right\}_{x=0};$$

$$\left[-\frac{\partial\Psi}{\partial x} + \frac{Bh}{G_3}\frac{\partial}{\partial y}\nabla^6 F\right]_{x=0} = 0. \tag{14}$$

Here $\gamma = \dfrac{D_{pk}}{RD^*}$.

Substituting into (14) the value of *Ci*, we obtain a system of equations in finite differences. Unknown, which belong to this system, must satisfy the conditions for *k=0* and *k=m*.

Conditions on the edges of a free-supported shell can be written as:

$$\eta_0 = \mu_0 = \zeta_0 = \xi_0 = \varphi_0 = \eta_m = \mu_m = \zeta_m = \xi_m = \varphi_m = 0 \qquad (15)$$

where *m-1* is the number of ribs that support the shell.

Solution of a last system we will found in the form:

$$\eta_k = A\sin\frac{ks\pi}{m};\;\; \mu_k = B\sin\frac{ks\pi}{m};\;\; \zeta_k = C\sin\frac{ks\pi}{m};\;\; \xi_k = M\sin\frac{ks\pi}{m};\;\; \phi_k = L\sin\frac{ks\pi}{m}; \qquad (16)$$

which satisfy boundary conditions (15) along an edges of the shell.

Here $1 \le s \le m-1$.

Equating determinant, which consists of coefficients at A, B, C, M, L, to zero we will get frequency equation of three-layered circular shell, which is supported by regular ring ribs, with hinge supporting of an edges. In the tables 1, 2 there are given values of parameter of first frequency of free vibrations mω of circular shell, supported by one and three ribs.

Table 1. Values of parameter m_ω for shell, which is supported by one annular rib

$\frac{a}{R}$	γ = 1			γ = 2			γ = 3		
	k_0			k_0			k_0		
	0,2	0,3	0,4	0,2	0,3	0,4	0,2	0,3	0,4
1,0	7,558	6,946	6,341	7,558	6,947	6,344	7,559	6,951	6,346
	7,551	6,939	6,330	7,551	6,939	6,340	7,551	6,948	6,337
2,0	7,559	6,948	6,344	7,560	6,948	6,344	7,561	6,954	6,347
	7,551	6,941	6,339	7,552	6,939	6,338	7,554	6,943	6,335
3,0	7,561	6,956	6,344	7,563	6,968	6,345	7,563	6,968	6,347
	7,552	6,950	6,337	7,555	6,962	6,335	7,551	6,959	6,329
4,0	7,565	6,958	6,345	7,566	6,971	6,345	7,567	6,971	6,348
	7,556	6,949	6,338	7,549	6,958	6,339	7,558	6,962	6,340

Table 2. Values of parameter m_ω for shell, which is supported by three annular ribs

$\frac{a}{R}$	γ = 1			γ = 2			γ = 2		
	k_0			k_0			k_0		
	0,2	0,3	0,4	0,2	0,3	0,4	0,2	0,3	0,4
1,0	7,052	6,463	5,922	7,058	6,465	5,928	7,068	6,467	5,932
	7,035	6,445	5,902	7,046	6,452	5,911	7,054	6,453	5,921
2,0	7,057	6,465	5,924	7,064	6,469	5,931	7,082	6,469	5,935
	7,043	6,453	5,909	7,048	6,451	5,920	7,071	6,452	5,924
3,0	7,068	6,468	5,932	7,069	6,473	5,940	7,089	6,472	5,943
	7,052	6,454	5,919	7,054	6,462	5,935	7,075	6,459	5,933
4,0	7,102	6,474	5,943	7,108	6,485	5,951	7,110	6,516	5,953
	7,092	6,467	5,929	7,083	6,464	5,940	7,102	6,507	5,934

Conclusions.

Thus, a mathematical model is constructed and an algorithm for investigating free vibrations of a three-layered circular shell, supported by annular ribs of rigidity, is developed. It is established that at number of ribs increase the frequency of free vibrations increases (or decreases) to a certain limit, after which the further number of ribs increase does not lead to an increase in the frequency vibrations of this form. At bending rigidity of ribs increase, the frequency of free vibrations increases to a certain limit, after which it remains constant and equal to frequency of free vibrations of the shell closed between ribs. At shear parameter increase frequency of free vibrations reduces and the Reisner's edge effect has no significant influence at any value of this parameter.

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