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The collective monograph is the result of the generalization of the conceptual work of scientists who consider current topics from such fields of knowledge as: management, management, technical sciences, law, ecology, information sciences and psychological sciences through the prism of international security studies. Content-functional lines and the key direction of the study of psycho- and sociogenesis of personality in age and pedagogical dimensions through the prism of revitalization are highlighted by each researcher in the context of the implementation of an individual sub-theme.

For scientists, educational staff, PhD candidates, masters of educational institutions, university faculties, stakeholders, managers and employees of management bodies at various hierarchical levels, and for everyone, who is interested in current problems of management, technical sciences, law, ecology, information sciences and psychological sciences through the prism of international security studies.

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SENSITIVITY ANALYSIS OF DYNAMIC SYSTEMS MODELS

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Abstract

The work is devoted to the study of the sensibility of the eigenvalues of dynamic systems models in state space and the stability of dynamic systems models. The problem of finding state estimates of dynamic systems is quite common in the design of optimal continuous and discrete control systems in their stochastic and deterministic form. The possibility of solving individual problems of finding estimates and optimal controls by projecting multidimensional spaces onto their own subspaces in order of increasing difficulty of solved problems has been considered.

The possibility to estimate the sensitivity of the parameters of linear dynamic system models using projection methods was accomplished. The study of dynamic systems models for sensitivity allowed us to identify critical changes in the eigenvalues of the system operator and predict unstable system operation modes. The solution of the stability problem of the dynamic system model is determined by the structure of the dynamic system model matrix, its rank, type, and multiplicity of the roots of the characteristic polynomial, and is solved by the theory of multiple roots of eigenvalues and eigenvectors based on Hershhorin's theorems. The stability of the dynamic system model is determined by the location of the eigenvalues on the complex plane.

Introduction

The problem of estimating the states of dynamic systems is quite common in the design of optimal continuous and discrete control systems in their stochastic and deterministic forms. Let's consider the possibility of solving individual problems of finding estimates and optimal control by the method of projection of multidimensional spaces onto their own subspaces in order of increasing difficulty of solved tasks. In the study of dynamic systems in individual cases, all output coordinates of the system can be directly measured and observed.

For linear systems that have such properties, the formation of an optimal control law as a function of state coordinates can be carried out even in the presence of measurement noise. However, in engineering practice it happens very often, that not all state coordinates can be observed and measured (for example, the speed of reaction in chemical plants) [12]. In these cases, the optimal control law is determined as a function of the best estimates of state coordinates, which are determined by measurements of the system's output signals. Thus, the problem of optimal control in a more general formulation includes both the problem of finding an optimal estimate of the system states and the problem of optimal control.

For modern control theory, the description of the system is characterized by the use of state variables and the application of projection methods that optimize its motion of controls in the space of possible states.

The most commonly used mathematical methods for control system design are:

–variational calculus;

–the principle of maximum;

–dynamic programming.

In all cases, the ultimate goal of design is to determine the optimal control law or control sequence that provides the maximum or minimum of a given functional that characterizes the quality of the system [3].

The general features of the three methods mentioned is the use of variational calculus: the first method has a direct relationship to the Euler-Lagrange equations, the second to the Hamilton principle, and the third to the Hamilton-Jacobi equations.

Euler-Lagrange Equations

$$\frac{d}{dt}(\frac{dL}{dq_i}) - \frac{dL}{dq_i} = 0 \ , \quad (1)$$

where

$$L = L(q_i, q_i) = T(q_i, q_i) - V(q_i); \quad (2)$$

where L – lagrangian;

q_i – generalized coordinates.

The Lagrange equation is derived from the Hamilton variational principle: any dynamical system will move under the action of conservative forces from any initial state so as to minimize the mean over time between the kinetic $T(q_i, q_i)$ and the potential $V(q_i)$ energies. The function that has the full energy of the system through generalized coordinates q and momenta p is called the Hamilton function.

$$H(\vec{p}, \vec{q}) = T_p + V; \quad (3)$$

[1] Сейдж Э.П., Мелса Дж. Л. (1974) Идентификация систем управления. М.: Наука, 284 с. [2] Ту Ю. (1971) Современная теория управления. М.: Машиностроение, 472 с.

[3] Сейдж Э. П., Уайт III Ч. С. (1982) Оптимальное управление системами. М.: Радио и связь, 392 с.

and

$$\frac{\partial p_i}{\partial t} = \frac{\partial H}{\partial q_i}; \quad \frac{\partial q_i}{\partial t} = \frac{\partial H}{\partial p_i}; \tag{4}$$

the Hamiltonian Canonical Equation.

Projection methods for assessing the states of a dynamic system

Let's consider the general solution to variational computation problems based on the method of projecting spaces onto subspaces, and evaluate the difficulties and advantages of this approach [45].

Let's assume that in a unidimensional or Euclidean space R, there is a given arbitrary vector $\vec{x}$ and some subspace S with the basis $\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_m$. The vector $\vec{x}$ can be represented (and uniquely) as a sum

$$\vec{x} = \vec{x}_S + \vec{x}_N, \ \ \vec{x}_S \in S, \ \ \vec{x}_N \perp S \tag{5}$$

where $\vec{x}_S$ – The orthogonal projection of the vector $\vec{x}$ onto the subspace S.

Orthogonality $\perp$ to the subspace S is understood to be orthogonality to all vectors in that subspace. Let us explain this with the figure 1.

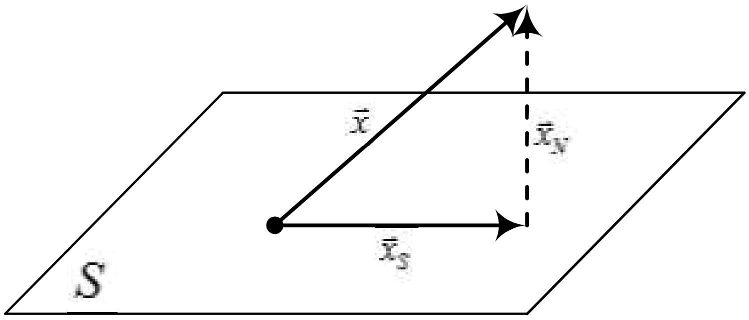

Figure 1 – Projection of $\vec{x}$ onto the subspace S [6]

To set up the arrangement (5) it's can be represented $\vec{x}_S$ in the form

$$\vec{x}_S = C_1\vec{x}_1 + C_2\vec{x}_2 + \cdots + C_m\vec{x}_m \ , \tag{6}$$

where $C_1, C_2, \ldots, C_m$ – some complex or real (for the Euclidean space) numbers. For Figure 1 $m = 2$.

To determine these numbers, we use the relations

$$(\vec{x} - \vec{x}_S, \vec{x}_k) = 0, \ \ k = 1,2, \ldots, m \tag{7}$$

Substituting the expression from (6) for $\vec{x}_S$ into (7)

$$\begin{cases} (\vec{x}_1\vec{x}_1)C_1 + \cdots + (\vec{x}_m\vec{x}_1)C_m + (\vec{x}\vec{x}_1)\,(-1) = 0 \\ \ldots \quad \ldots \quad \ldots \quad \ldots \\ (\vec{x}_1\vec{x}_m)C_1 + \cdots + (\vec{x}_m\vec{x}_m)C_m + (\vec{x}\vec{x}_m)(-1) = 0 \\ \vec{x}_1 C_1 + \cdots + \vec{x}_m C_m + \vec{x}_S\,(-1) = 0 \end{cases} \tag{8}$$

[4] Гантмахер Ф.Р. (2004) Теория матриц. М.: ФИЗМАТЛИТ, 560 с.

[5] Димова Г.О. (2020) Методи і моделі упорядкування експериментальної інформації для ідентифікації і прогнозування стану безперервних процесів: монографія. Херсон: Видавництво ФОП Вишемирський В.С., 176 с.

[6] Гантмахер Ф.Р. (2004) Теория матриц. М.: ФИЗМАТЛИТ, 560 с.

Considering this system of equations as a system of linear homogeneous equations, which have zero solutions $C_1, C_2, \dots, C_m, -1$, its determinant can be equated to zero (having previously transposed it relative to the principal diagonal)

$$\begin{vmatrix} (\vec{x}_1\vec{x}_1) & \cdots & (\vec{x}_1\vec{x}_m) & \vec{x}_1 \\ \dots & \dots & \dots & \dots \\ (\vec{x}_m\vec{x}_1) & \cdots & (\vec{x}_m\vec{x}_m) & \vec{x}_m \\ (\vec{x}\vec{x}_1) & \cdots & (\vec{x}\vec{x}_m) & \vec{x}_S \end{vmatrix} = 0 \qquad (9)$$

By extracting the member of this determinant that contains $\vec{x}_S$, we can get

$$\vec{x}_S = \frac{\begin{vmatrix} & \Gamma & & \begin{matrix} \vec{x}_1 \\ \cdots \\ \vec{x}_m \end{matrix} \\ (\vec{x}\vec{x}_1) & \cdots & (\vec{x}\vec{x}_m) & 0 \end{vmatrix}}{\Gamma} \qquad (10)$$

where $\Gamma = \Gamma(\vec{x}_1\vec{x}_2 \dots \vec{x}_m)$ – the Gram determinant for the vectors $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m$ (due to the independence of these vectors $\Gamma \neq 0$).

With (5)

$$\vec{x}_N = \vec{x} - \vec{x}_S = \frac{\begin{vmatrix} & \Gamma & & \begin{matrix} \vec{x}_1 \\ \cdots \\ \vec{x}_m \end{matrix} \\ (\vec{x}\vec{x}_1) & \cdots & (\vec{x}\vec{x}_m) & 0 \end{vmatrix}}{\Gamma}. \qquad (11)$$

The formulas (10) and (11) express the projection $\vec{x}_S$ of the vector $\vec{x}$ onto the subspace S, observed at the output (dimensional vector parameters of the process running in the system), by linear combinations of which we will find (restore) estimates of the vectors of the state space of the system [7].

Let $\vec{y}$ be an arbitrary vector in the set of vectors in S, and $\vec{x}$ – an arbitrary vector in R. If we construct the vectors from the initial coordinates, then $|\vec{x} - \vec{y}|$ and $|\vec{x} - \vec{x}_S|$ will respectively equal the lengths of the inclined and altitude drawn from the end of the vector $\vec{x}$ to the surface S (fig. 1). Therefore, by considering the altitude being smaller than the inclined, we have $h = |\vec{x} - \vec{x}_S| \leq |\vec{x} - \vec{y}|$ (the equality sign will only be when $\vec{y} = \vec{x}_S$). Thus, among all the vectors $\vec{y} \in S$, the vector $\vec{x}_S$ is the least inclined from the given vector $\vec{x} \in R$. The value $h = \sqrt{(\vec{x} - \vec{x}_S)(\vec{x} - \vec{x}_S)}$ of the given vector $\vec{x} \in R$ is the quadratic error in approximating $\vec{x} \approx \vec{x}_S$.

Projection methods of investigation allow us to simultaneously and independently solve the problem of evaluating the state vectors of a dynamic system and finding optimal control sequences.

Let's apply this approach to solve the problem of controlling a multi-dimensional system with coordinates that are not accessible for observation.

[7] Гантмахер Ф.Р. (2004) Теория матриц. М.: ФИЗМАТЛИТ, 560 с.

Research on the sensitivity of eigenvalues of dynamic systems model matrices in the state space

When applying the generalized approach outlined in point 1 to solve the problem of controlling a multi-dimensional system with coordinates inaccessible for observation, only output signals can be measured directly.

The measured coordinates are treated as output variables and are denoted by $y_1, y_2, \dots, y_p$, considering them components of the vector $\vec{y}$.

When solving the problem, the output variables are assumed as linear functions of the state coordinates $\vec{x}(k)$ and are related to them through a linear transformation

$$\vec{y}(k) = \mathbf{M}\vec{x}(k), \qquad (12)$$

where $\vec{x}$ – n-dimensional vector;

$\vec{y}$ – p-dimensional vector;

$\mathbf{M}$ – $p \times n$ matrix with $p \leq n$.

In the case where the output vector size is smaller than the state vector, the matrix $\mathbf{M}$ is rectangular and has no inverse matrix. In fact, this matrix is the output matrix (matrix of measured variables) [8][9][46, 119, 124].

When researching the possibilities of optimal control, we will start from the fact that the system is described by a vector-matrix differential equation [10][90, 120, 121, 124].

$$\vec{x} = \mathbf{A}(t)\vec{x}(t) + \mathbf{B}(t)\vec{u}(t) + \vec{n}(t)\ , \qquad (13)$$

where $\vec{x}(t)$ – n-dimensional vector that represents the state variables;

$\vec{u}(t)$ – k-dimensional vector that represents the control influences;

$\vec{n}(t)$ – s- dimensional vector that represents external random influences;

$\mathbf{A}(t)$ – matrix of process coefficients that occur in the system;

$\mathbf{B}(t)$ – control matrix.

The solution of equation (13) has the form [11][12]

$$\vec{x}(t) = \mathbf{\Theta}(t, t_0)\vec{x}(t_0) + \int_{t_0}^{t}[\mathbf{\Theta}(t,\tau)\mathbf{B}(\tau)\vec{u}(\tau) + \vec{n}(\tau)]d\tau, \qquad (14)$$

where $\Theta(t, t_0)$ – the transition matrix that satisfies a homogeneous differential equation

$$\frac{d\mathbf{\Theta}(t, t_0)}{dt} = \mathbf{A}(t)\mathbf{\Theta}(t, t_0) \qquad (15)$$

[8] Димова Г.О. (2017) Аналіз чутливості власних значень матриць моделей динамічних систем. *Вимірювальна та обчислювальна техніка в технологічних процесах: Матеріали XVII Міжнар. наук.-техн. конференції* (8-13 червня 2017 р., м.Одеса); Одес. нац. акад. зв'язку ім. О.С.Попова. Одеса–Хмельницький: ХНУ, С. 150-152.

[9] Ту Ю. (1971) Современная теория управления. М.: Машиностроение, 472 с.

[10] Марасанов В.В., Забытовская О.И., Дымова А.О. (2012) Прогнозирование структуры динамических систем. *Вісник ХНТУ*, № 1 (44), С. 292-302.

[11] Марасанов В.В., Дымова А.О., Дымов В.С. (2016) Исследование на чувствительность моделей динамических систем, полученных проекционным методом. *Проблеми інформаційних технологій*, Херсон, №1(019). С. 169-173.

[12] Димова Г.О. (2020) Методи і моделі упорядкування експериментальної інформації для ідентифікації і прогнозування стану безперервних процесів: монографія. Херсон: Видавництво ФОП Вишемирський В.С., 176 с.

and relationship

$$\mathbf{\Theta}(t_0, t_0) = \mathbf{I}, \tag{16}$$

where **I** – unit matrix.

In discrete dynamic systems with digital control $u(\tau) = u(kT)$ for $kT \leq \tau \leq (k+1)T$, the solution in discrete form is given by the state transition equation

$$\vec{x}(k+1) = \mathbf{\Theta}(k)\vec{x}(k) + \mathbf{G}(k)\vec{\vec{u}}(k) + \vec{\vec{\vec{m}}}(k)\ , \tag{17}$$

where

$$\mathbf{\Theta}(k) = \mathbf{\Theta}((\overline{k+1})T, kT), \tag{18}$$

$$\boldsymbol{G}(k) = \int_{kT}^{(k+1)T} \mathbf{\Theta}\,((\overline{k+1})T, \tau)\,\mathbf{B}(\tau)d\tau, \tag{19}$$

$$\vec{\vec{\vec{m}}}(k) = \int_{kT}^{(k+1)T} \mathbf{\Theta}((k+1)T, \tau)n(\tau)d\tau, \tag{20}$$

The principle of constructing optimal controls for a dynamic system is also determined by a quality index, which takes into account constraints that guarantee the physical realization of an optimal control for the dynamic system. When implementing digital control systems, the quality index is determined by a quadratic form.[13][14]

$$J_N = \sum_{k=1}^{N} \{[\vec{x}^{d}(k) - \vec{x}(k)]'\mathbf{Q}(k)[\vec{x}^{d}(k) - \vec{x}(k)] + \lambda \vec{u}'(k-1)\mathbf{H}(k-1)\vec{u}(k-1)\}, \tag{21}$$

where $\vec{x}^d(k)$ – vector of the desired state;

Q, **H** – positively defined symmetric matrices;

λ – constant multiplier.

The first addendum in (21) gives the deviation from the process specified at any time kT, the second addendum takes into account the energy limitations of the controlling influence [15].

By selecting the appropriate elements of the matrix **Q**, any state coordinate of the process can be made more important and effective for assessing the system's quality compared to another variable. Similarly, by selecting elements of the matrix **H**, desired limitations can be imposed on the energy of controlling influences. Optimal control involves determining a sequence of control vectors $\vec{\vec{u}}'(0), \vec{\vec{u}}'(1), \dots, \vec{\vec{u}}'(N-1)$ that minimize the expected mean of the quality indicator [16].

[13] Сейдж Э.П., Мелса Дж. Л. (1974) Идентификация систем управления. М.: Наука, 284 с.

[14] Ту Ю. (1971) Современная теория управления. М.: Машиностроение, 472 с.

[15] Марасанов В.В., Дымова А.О., Дымов В.С. (2016) Проекционные методы оценки состояний динамической системы при частично наблюдаемых выходных координатах. *Проблеми інформаційних технологій*, Херсон, №1(019). С. 259-264.

[16] Марасанов В.В., Дымова А.О., Дымов В.С. (2016) Исследование на чувствительность моделей динамических систем, полученных проекционным методом. Проблеми інформаційних технологій, Херсон, №1(019). С. 169-173.

For linear systems described by state equations (17), the control vector that minimizes the expected mean of the quality indicator is given by the formula [17][18]:

$$\hat{\vec{u}}(k/k) = \mathbf{D}(N-k)\hat{\vec{x}}(k/k)\ , \qquad (22)$$

where

$\mathbf{D}(N-k)$ – The feedback matrix, whose elements are the feedback coefficients (it changes over time, as it is calculated at each step);

$\hat{\vec{x}}(k/j)$ – estimation of the state vector $\vec{x}(k)$, which uses measured values

$$\vec{y}(j), \vec{y}(j-1), \dots, \vec{y}(0) \qquad (23)$$

output vectors, optimum in the sense that the expected mean value is minimized

$$E\,\{[\vec{x}(k) - \hat{\vec{x}}(k/j)]'[\vec{x}(k) - \hat{\vec{x}}(k/j)]\} \qquad (24)$$

where according to formula (5) and Fig. 1 $\hat{\vec{x}}(k/k)$ is the orthogonal projection of the state vector onto the subspace $Y(j)$. Therefore

$$\vec{x}(k) = \hat{\vec{x}}(k/k) + \tilde{\vec{x}}(k/k), \qquad (25)$$

$Y(j)$ is a subspace of the space $\vec{X}(k)$ – the space of the state vectors of the dynamic system.

According to (25), the vector of optimal control $\hat{\vec{u}}(k/k)$ can be written as the sum of its orthogonal projection $\hat{\vec{u}}(k/k)$ onto the subspace $Y(j)$ and its normal component $\tilde{\vec{u}}(k/k)$ [19].

According to formula (5) and Fig. 1, by analogy it can be written

$$\hat{\vec{u}}(k/k) + \tilde{\vec{u}} = \mathbf{D}(N-k)\hat{\vec{x}}(k/k) + \mathbf{D}(N-k)\tilde{\vec{x}}(k/k). \qquad (26)$$

Using the basic properties of orthogonal projection [20], it`s find

$$\begin{aligned} \hat{\vec{u}}(k/k) &= \mathbf{D}(N-k)\hat{\vec{x}}(k/k) \\ \tilde{\vec{u}}(k/k) &= \mathbf{D}(N-k)\tilde{\vec{x}}(k/k) \end{aligned} \qquad (22')$$

The orthogonal projection of $\hat{\vec{u}}(k/k)$ which is the best estimate for $\vec{u}^{\circ}(k)$ is linearly related to the best estimate for $\vec{x}(k)$. The normal component of the vector $\tilde{\vec{u}}(k)$ constitutes the estimation error [21]. The estimation of $\vec{u}^{\circ}(k)$ can be physically realized as an estimation function $\hat{\vec{x}}(k/k)$ that can be determined by measuring the output signals.

Let us now show that using the principle of optimality and when the optimal control vector $\vec{u}^{\circ}(k)$ is replaced by its best estimate (22), the quality of the system is determined by the minimum mean value of J_N. The solution is based on dynamic

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[17] Димова Г.О. (2020) Методи і моделі упорядкування експериментальної інформації для ідентифікації і прогнозування стану безперервних процесів: монографія. Херсон: Видавництво ФОП Вишемирський В.С., 176 с.

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[19] Ланкастер П. (1978) Теория матриц. М.: Наука, 280 с.

[20] Гантмахер Ф.Р. (2004) Теория матриц. М.: ФИЗМАТЛИТ, 560 с.

[21] Димова Г.О. Дослідження чутливості та стійкості моделей динамічних систем. *Комп'ютерно-інтегровані технології: освіта, наука, виробництво.* Луцьк. 2017. № 28-29. С. 55-59.

programming. Expression (21) describes the optimal control law [22]. To prove this, the symmetry of the matrices $\mathbf{Q}$ and $\mathbf{H}$ is used [23]. Let's denote the minimum expected mean value of J_N by

$$f_N[\vec{x}(0)] = \min_{\vec{u}(j)} EJ_N, \quad j = 1,2,\dots,N. \tag{27}$$

It is obvious that when $\vec{u}(j) = \vec{u}^\circ(j)$, then $EJ_N = f_N$ and $EJ_N - f_N = 0$. However, when $\vec{u}(k) = \vec{u}^\circ(k)$, then $EJ_N - f_N > 0$, that is, an error is introduced since by definition f_N is the minimum for EJ_N. Thus, the problem is to determine an estimate for $\vec{u}^\circ(k)$ that minimizes the error $EJ_N - f_N$ due to the non-realization of $\vec{u}^\circ(k)$. This estimate is called the best estimate and it is given by the orthogonal projection of $\vec{u}\,(k/k)$ and therefore equation (22) determines the optimal control law for processes with coordinates inaccessible for measurement. The task reduces to the finding of estimates for a multi-step process in which the estimates are successively found for all steps and optimal solutions found in the previous step are used in each successive step, that is, the principle of dynamic programming is implemented [24].

Using the principle of optimality, the minimum value of $f_N[\vec{x}(0)]$ for an N-step control process with $N > 1$ can be written as

$$f_N[\vec{x}(0)] = \min_{\vec{u}(0)} E\{\vec{x}'(1)\mathbf{Q}(1)\vec{x}(1) + \lambda\vec{u}'(0)\mathbf{H}(0)\vec{u}(0) + f_{N-1}[\vec{x}(1)]\}, \tag{28}$$

where the connection between $\vec{x}(1)$ and $\vec{u}(0)$ is given by the equation

$$\vec{x}(k+1) + \phi(k)\vec{x}(k) + G(k)\vec{u}(k) + \vec{m}(k). \tag{29}$$

For $N = 1$ the minimum is equal to [25]

$$f_1[\vec{x}(0)] = \min_{\vec{u}(0)} E\{\vec{x}'(1)\mathbf{Q}(1)\vec{x}(1) + \lambda\vec{u}'(0)\mathbf{H}(0)\vec{u}(0)\}. \tag{30}$$

Here, as mentioned before, the symmetry of the matrices $\mathbf{Q}$ and $\mathbf{H}$ is assumed.

In equation (13) $\mathbf{A}$ is the main matrix of the system, since its structure determines the character of the state transition matrix (15) and therefore the sensitivity of the roots of the characteristic equation of the matrix $\mathbf{A}$ determines the sensitivity of the system to disturbances. Previously in [26], an estimate of the matrix $\mathbf{A}$ (formula (13)) was found using the factorization of the correlation functions of a set of output parameters of the identified dynamic system. The simplest method of sensitivity analysis is numerical study of the parameter model of the system in the full range of changes of the determinant set of parameters. The main method of research in sensitivity theory is the use of sensitivity functions.

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[25] Димова Г.О. (2020) Методи і моделі упорядкування експериментальної інформації для ідентифікації і прогнозування стану безперервних процесів: монографія. Херсон: Видавництво ФОП Вишемирський В.С., 176 с.

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Let $\lambda_1, \lambda_2, \ldots, \lambda_m$ be a set of the eigenvalues of the matrix **A**. Then the state variables $\vec{x}_i$, $i = 1, n$ and quality indices $J_1, J_2, \ldots, J_s$ are unique functions of the parameters $\lambda_1, \lambda_2, \ldots, \lambda_m$, that is

$$\vec{x}_i(t, \lambda) = \vec{x}(t, \lambda_1, \lambda_2, \ldots, \lambda_m), \quad i = 1, n \tag{31}$$

and

$$J_i(\lambda) = J_i(\lambda_1, \lambda_2, \ldots, \lambda_m), \quad i = \overline{1, s} \tag{32}$$

The partial derivatives $\frac{\partial x_i(t,\lambda)}{\partial \lambda k}$, $\frac{\partial J_i(\lambda)}{\partial \lambda k}$ are called the first-order sensitivity functions of the variables $\vec{x}_i$ and J_i for the arguments $\lambda_1, \lambda_2, \ldots, \lambda_m$

$$\frac{\partial^k \vec{x}_i}{\partial \lambda_1^{k1} \partial \lambda_2^{k2} \ldots \partial \lambda_m^{km}}, \quad \frac{\partial^k J_i}{\partial \lambda_1^{k1} \partial \lambda_2^{k2} \ldots \partial \lambda_m^{km}}, \quad k_1 + k_2 + \ldots + k_m = k \tag{33}$$

are called k-th order sensitivity functions for their respective combinations of parameters. It is obvious that the sensitivity functions of the state variables $\vec{x}(t, \lambda)$ depend on *t* and parameters λ, while the sensitivity functions of the quality indices only depend on parameters $\lambda_1, \ldots, \lambda_m$. The sensitivity functions of different orders are solutions of the model equations of the system. These equations are the sensitivity equations.

The ensemble of the output mathematical model (13) and the quality criteria (21) that define the sensitivity functions are called the system's sensitivity model under study.

Let's consider two cases. The first: $(\overline{\lambda_1}, \ldots, \overline{\lambda_m})$ is a fixed (calculated) parameter value; $(\overline{\lambda_1}, \ldots, \overline{\lambda_m}) = \overline{\lambda}$ is the base set; $\overline{\lambda}$ corresponds to the set of state variables [76, 114, 115]. $\overline{\vec{x}_i} = \vec{x}_i(t, \lambda)$ will be called the main basic motion of the system. The basic motion corresponds to the basic values of the quality indices $\overline{J_i} = J_i(\overline{\lambda})$. The second: when the parameters $\lambda_i = \overline{\lambda_i} + \mu$ are changed, we get a new motion $\vec{x}_i = \vec{x}_i(t, \overline{\lambda_1} + \mu_1, \ldots, \overline{\lambda_m} + \mu_m) = \vec{x}_i(t, \overline{\lambda} + \vec{\mu})$, corresponding to new values of the quality indices $J_i = J_i(\overline{\lambda_1} + \mu_1, \ldots, \overline{\lambda_m} + \mu_m) = J_i(\vec{\lambda} + \vec{\mu})$. The vector $\Delta\vec{x}_i = \vec{x}_i(t, \overline{\vec{\lambda}} + \vec{\mu}) - \vec{x}_i(t, \overline{\vec{\mu}})$, is called additional motion induced by the change of the eigenvalues of the matrix **A**. There can be two outcomes:

1) the real part of one or more eigenvalues of the characteristic polynomial is found to be positive and the system will be unstable;

2) the total change of λ_i will lead to a change in the quality index that does not meet the project requirements.

The analysis of tasks 1) and 2) will be carried out by means of chaos theory based on the Hershhorin's theorem [27].

Let the matrix $\mathbf{A} = \|a_{ij}\|$, $i, j = \overline{1, n}$ obtained in [28] be a constant square matrix with real elements. The matrix $\|\lambda \mathbf{I} - \mathbf{A}\|$, where λ is a scalar independent variable, will be a characteristic matrix of the identified system and its determinant

[27] Димова Г.О. (2020) Методи і моделі упорядкування експериментальної інформації для ідентифікації і прогнозування стану безперервних процесів: монографія. Херсон: Видавництво ФОП Вишемирський В.С., 176 с.

[28] Димова Г.О. Дослідження чутливості та стійкості моделей динамічних систем. *Комп'ютерно-інтегровані технології: освіта, наука, виробництво.* Луцьк. 2017. № 28-29. С. 55-59.

$$\det(\lambda\mathbf{I}-\mathbf{A})=\lambda^n+a_1\lambda^{n-1}+\cdots+a_n \quad (34)$$

– will be the characteristic polynomial of the matrix **A**.

Consider the case when all the eigenvalues (roots of the polynomial (34)) are different and depend continuously on all elements of the matrix **A**. Let`s define the perturbation of the matrix **A** in the form

$$\mathbf{A}(\Delta_\lambda)=\mathbf{A}+\Delta_\lambda\mathbf{B}, \quad (35)$$

where **B** – a random real square matrix of the same order as the matrix **A**.

The value of Δ_λ is determined based on the level of noise at the output of the identified system. From the theory of perturbations of linear operators and matrices [29], it follows that $\lambda_i(\Delta_\lambda)$ – the eigenvalues of the matrix $\mathbf{A}+\Delta_\lambda\mathbf{B}$ and the corresponding eigenvectors $\mathbf{x}_i(\Delta_\lambda)$ are continuous and differentiable functions of the parameter Δ_λ:

$$\lambda_i(\Delta_\lambda)=\lambda_i+\sum_{j=1}^{\infty}\frac{1}{j!}\left(\frac{\partial^j\lambda_i(\Delta_\lambda)}{\partial\Delta_\lambda^j}\right)\Big|_{\Delta_\lambda=0}\Delta_\lambda^j, \quad (36)$$

$$\mathbf{x}_i(\Delta_\lambda)=\mathbf{x}_i+\sum_{j=1}^{\infty}\frac{1}{j!}\left(\frac{\partial^j\mathbf{x}_i(\Delta_\lambda)}{\partial\Delta_\lambda^j}\right)\Big|_{\Delta_\lambda=0}\Delta_\lambda^j. \quad (37)$$

In formulas (3.36) and (3.37), $\beta_{ij}=\frac{\partial^j\lambda_i(\Delta_\lambda)}{\partial\Delta_\lambda^j}\Big|_{\Delta_\lambda=0}$ – the sensitivity coefficient of j-th order, $\boldsymbol{\gamma}_{ij}=\frac{\partial^j\mathbf{x}_i(\Delta_\lambda)}{\partial\Delta_\lambda^j}\Big|_{\Delta_\lambda=0}$ – the sensitivity vector of j-th order.

According to Hershhorin's theorem [30], any eigenvalue of the matrix **A** lies at least in one circle with center at a_{ii} and radius $\rho=\sum_{j\neq i}|a_{ij}|$. If the sum

$$a_{ii}+\rho>0, \quad (38)$$

then the system model for one i-th eigenvalue (or several eigenvalues) is unstable. This is possible even with negative a_{ii}. Under the condition $|a_{ij}|<\rho$, which is fulfilled according to formulas (36) and (38) at $\lambda_i(\Delta_\lambda)>0$, which must be checked for all $i=\overline{1,n}$ ($n\times n$ is the order of the obtained estimate of the dynamic system model matrix) [90], and in this case it is necessary to find permissible additional movements of the system and again check the stability of the matrix **A** of the dynamic model. The solution of this problem is reduced to filtering the measured output n^* parameters ($n^*<n$).

In general case, the eigenvalues of the matrix **A** are complex numbers i, if the real part of the complex number eigenvalue is positive at least for one λ_i, then the obtained estimate of the matrix **A** of the dynamic system model will be unstable.

Projection methods for studying dynamic systems allow, with a certain selection of the matrices **Q** and **H**, to solve the problem of finding a quasi-optimal control with a certain precision when observing output signals of the system incompletely.

[29] Розенвассер Е.Н., Юсупов Р.М. (1981) Чувствительность систем управления. М.: Наука, 464 с.
[30] Уилкинсон Дж.Х. (1972) Алгебраическая проблема собственных значений. М.: Наука, 565 с.

Studies of the obtained dynamic systems models for sensitivity allow to determine critical changes in the eigenvalues of the system operator and predict unstable modes of system operation.

3 Researching the sensitivity and stability of dynamic system models

When solving the identification problem of a multi-dimensional dynamic system, it was assumed that the model of its dynamics can be described by a vector-matrix differential equation (13).

The matrix $\mathbf{A}(t)$ was estimated based on factorization of the correlation matrix of the system output signals [31], and the vector of control $\vec{u}(t)$ was determined by the projection method [32], assuming that the system has an optimal control and $\vec{n}(t)$ is a multi-dimensional white noise, the power of which depends on the operation modes of the dynamic system.

The final stage is to study the sensitivity of the obtained model to disturbances and its stability.

Solving the task of this step requires taking into account that the elements of the matrix $\mathbf{A}(t)$ are determined from experiment and therefore may have errors. In this case, the matrix $\mathbf{A}(t)$, which estimate was obtained during solution of factorization problem of the system output processes correlation function [33], is an approximation of the matrix corresponding to exact measurements.

Considering the accuracy characteristics of the measurement system and the noise level, the amount of error ε of the elements a_{ij} of the matrix $\mathbf{A}(t)$ is $\varepsilon \leq \delta$. This can be represented as the sum of two matrices in the form of the actual system matrix: $\mathbf{A}(t)$ – the real calculated matrix without taking into account the noise and the matrix $\mathbf{E}$ – of the same order as the matrix $\mathbf{A}$, and looking at the matrix $(\mathbf{A}(t) + \mathbf{E})$. Elements of the matrix $\mathbf{E}$:

$$|e_{ij}| \leq \varepsilon. \qquad (39)$$

The full solution to the posed problem can be reduced to an algebraic problem of the matrices $\mathbf{A}(t)$ and $(\mathbf{A}(t) + \mathbf{E})$ eigenvalues and consist not only in determining the eigenvalues and eigenvectors of the matrices $\mathbf{A}(t)$ and $(\mathbf{A}(t) + \mathbf{E})$, but also in estimating the possible variations of the eigenvalues of all matrices of the class $(\mathbf{A} + \mathbf{E})$ that satisfies condition (39). In this case, the fundamental algebraic problem of eigenvalues is to determine the values λ in the equation

$$\mathbf{A}x = \lambda x \qquad (40)$$

the system of n homogeneous linear equations with n unknowns. Equation (40) can be represented in the form

[31] Марасанов В.В., Забытовская О.И., Дымова А.О. (2012) Прогнозирование структуры динамических систем. Вісник ХНТУ, № 1 (44), С. 292-302.

[32] Марасанов В.В., Дымова А.О., Дымов В.С. (2016) Проекционные методы оценки состояний динамической системы при частично наблюдаемых выходных координатах. *Проблеми інформаційних технологій*, Херсон, №1(019). С. 259-264.

[33] Димова Г.О. (2020) Методи і моделі упорядкування експериментальної інформації для ідентифікації і прогнозування стану безперервних процесів: монографія. Херсон: Видавництво ФОП Вишемирський В.С., 176 с.

$$(\mathbf{A} - \lambda\mathbf{I})x = 0, \tag{41}$$

where $\mathbf{I}$ – $(n\times n)$ unit matrix.

For any λ the system of equations (41) has only the solution $x = 0$. A nontrivial solution exists when the matrix $(\mathbf{A} - \lambda\mathbf{I})$ is singular, that is, when its determinant

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0, \tag{42}$$

Expanding the determinant in the left side of equation (42) by degrees of λ, we obtain

$$a_0 + a_1\lambda + \cdots + a_{n-1}\lambda^{n-1} + (-1)^n\lambda^n = 0 \tag{43}$$

– characteristic equation and characteristic polynomial. In the complex field, this equation always has n roots, which can be complex even with a real matrix $\mathbf{A}$ and any multiplicity, up to n [34]. They are called eigenvalues or characteristic numbers. Each value λ corresponds to at least one non-trivial solution x. If the rank of the matrix $(\mathbf{A} - \lambda\mathbf{I})$ is less than $(n-1)$, then there will be no less than two linearly independent vectors that satisfy the equation (41). If x is a solution of equation (41), then kx is also a solution for any k. Even if the rank of $(\mathbf{A} - \lambda\mathbf{I})$ is equal to $(n-1)$, the eigenvector corresponding to λ, determined to an arbitrary multiplier, is normalized. The most convenient methods for normalization are:

а) the sum of the squares of the moduli of the vector $\mathbf{x}$ components is equal to one;

б) the largest in modulus component of vector $\mathbf{x}$ is equal to one;

в) the sum of the moduli of the vector $\mathbf{x}$ components is equal to one.

Methods a) and b) are rarely used for normalization.

Formulas (39-43) are also valid for the transposed matrix $\mathbf{A}^T$. In this case, there are the following relationships:

$$\begin{gathered} \mathbf{A}^T\mathbf{y} = \lambda\mathbf{y}; \qquad \det(\mathbf{A}^T - \lambda\mathbf{I}) = 0; \\ \mathbf{A}^T\mathbf{y}_i = \lambda_i\mathbf{y}_i; \quad \mathbf{y}_i^T\mathbf{A} = \lambda_i\mathbf{y}^T; \\ \mathbf{x}_i^T\mathbf{y}_j = 0 \qquad (\text{якщо } \lambda_i \neq \lambda_j)., \end{gathered} \tag{44}$$

Note that since $\mathbf{x}_i$ and $\mathbf{y}_j$ can be complex vectors, $(\mathbf{x}_i^T\mathbf{y}_j)$ is not a scalar product in the usual sense. Indeed, we have

$$\mathbf{x}_i^T\mathbf{y}_j = \mathbf{y}_j^T\mathbf{x}_i, \tag{45}$$

and not

$$\mathbf{x}_i^T\mathbf{y}_j = \overline{\mathbf{y}_j^T\mathbf{x}_i}. \tag{46}$$

If $\mathbf{x}$ is complex, then it may happen that

$$\mathbf{x}^{\mathrm{T}}\mathbf{x} = 0, \tag{47}$$

while this scalar product is always positive for all non-zero $\mathbf{x}$.

The solution to the problem of stability of the model of the dynamic system (13) in the general case is determined by the structure of the matrix $\mathbf{A}$, its rank, type and

[34] Димова Г.О. (2020) Методи і моделі упорядкування експериментальної інформації для ідентифікації і прогнозування стану безперервних процесів: монографія. Херсон: Видавництво ФОП Вишемирський В.С., 176 с.

multiplicity of the characteristic polynomial roots and can be solved by the method of perturbation theory of various orders of eigenvalues and eigenvectors [35].

There are three approaches to solving the stability problem of the model (13):

1) based on algebraic functions and the theory of vector and matrix norms;

2) based on complex argument functions, following the Cauchy's theorem – the principle of argument;

3) based on Hershhorin's theorems.

The first and second approaches in terms of the characteristic polynomial for multiple values λ are too coarse, since the polynomial does not sufficiently reflect the structure of the matrix $(\mathbf{A} + \varepsilon\,\mathbf{E})$. Therefore, we use the third approach – based on the theorem of Hershhorin. It has greater practical value and takes into account the structure of the matrices **A** and $(\mathbf{A} + \varepsilon\,\mathbf{E})$, which follows from the theorem 1 [36].

Theorem 1. Any eigenvalue of the matrix **A** lies in at least one circle with center a_{ii} and radii

$$r_i = \sum_{j \neq i} |a_{ij}| \tag{48}$$

on the complex plane.

It means that for any eigenvalue λ there is at least one non-zero **x** such that $\mathbf{Ax} = \lambda\mathbf{x}$. In the selected **x**, let's normalize (2) by the *r*-th largest by the modulus component of the vector **x**:

$$\mathbf{x}^T = (x_1, x_2, \dots, x_{r-1}, 1, x_{r+1}, \dots, x_n),$$

where $|x_i| \leq 1, i \neq r$.

Therefore:

1)

$$\sum_{j=1}^{n} a_{rj} x_j = \lambda x_r = \lambda \tag{49}$$

2)

$$|\lambda - a_{rj}| \leq \sum_{j \neq r} |a_{rj} x_j| \leq \sum_{j \neq r} |a_{rj}||x_j| \leq \sum_{j \neq r} |a_{rj}| \tag{50}$$

and λ lies in one of these circles.

Theorem 2 [37]. If φ circles of the *theorem 1* form a connected region, then in this connected region there are exactly φ eigenvalues of the matrix **A**. From the proof of the theorem and the theory of algebraic functions it follows that the roots of the characteristic polynomial for the matrix $(\mathbf{A} + \varepsilon\,\mathbf{E})$ are continuous functions of the perturbation ε and radii $\varepsilon\, r_i$ for ε from 0 to 1 and at $\varepsilon = 1$. Similar conclusions and results can be obtained if the transposed matrix $\mathbf{A}^T$ is considered instead of the matrix **A**.

To determine the stability of the model, it is necessary to analyze the eigenvalues of the matrix $(\mathbf{A} + \varepsilon\,\mathbf{E})$, using its Jordan canonical form. For simple eigenvalues λ_i of

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[35] Деруссо П., Рой Р., Клоуз Ч. (1970) Пространство состояний в теории управления. М.: Наука, 620 с.

[36] Ланкастер П. (1978) Теория матриц. М.: Наука, 280 с.

[37] Ланкастер П. (1978) Теория матриц. М.: Наука, 280 с.

the matrices that have elementary divisors, the matrices **A** and $(\mathbf{A} + \varepsilon\, \mathbf{E})$, are diagonal. Therefore, there exists a matrix **H** such that $(\mathbf{H}^{-1}\mathbf{AH}) = \text{diag}\,(\lambda_i)$, where columns of **H** form a full system of right eigenvectors x_i, and rows of $\mathbf{H}^{-1}$ form a full system of left eigenvectors $\mathbf{y}_j^T$.

Upon normalizing $||\boldsymbol{x}||_2 = ||\boldsymbol{y}||_2 = 1$ [47, 48, 79, 127] ($||\bullet||_2$ – Euclidean norm), we can take the *i*-th column of the matrix **H** as the eigenvector $\mathbf{x}_i$ and then the *i*-th row of the matrix $\mathbf{H}^{-1}$ will be

$$\mathbf{y}_i^T/\mathbf{s}_i, \tag{51}$$

where $\mathbf{s}_i = \mathbf{y}_i^T\mathbf{x}_i$ at $i = (1, 2, \dots, n)$.

Then

$$\mathbf{H}^{-1}(\mathbf{A} + \varepsilon\mathbf{E})\mathbf{H} = \text{diag}\,\lambda + \varepsilon \begin{vmatrix} \beta_{11}/s_1 & \beta_{12}/s_1 & \dots & \beta_{1n}/s_1 \\ \beta_{21}/s_2 & \beta_{22}/s_2 & \dots & \beta_{2n}/s_2 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n1}/s_n & \beta_{n2}/s_n & \dots & \beta_{nn}/s_n \end{vmatrix}. \tag{52}$$

According to Hershhorin's *theorem 1*, we obtain that the eigenvalues lie in circles with centers $\lambda_i + \varepsilon\beta_i/s_i$ and radii $\sum_{j\neq i}|\beta_{ij}/s_i|$, where $\beta_{ij} = \mathbf{y}_i^T\mathbf{E}\mathbf{x}_j$, since $||\mathbf{E}||_2 \leq n$, then

$$|\beta_{ij}| = |\mathbf{y}_i^T\mathbf{E}\mathbf{x}_j| \leq \|\mathbf{y}_i\|_2\|\mathbf{E}\mathbf{x}_j\|_2 \leq \|\mathbf{E}\|_2 \leq \|\mathbf{E}\|_2\|\mathbf{y}_i\|_2\|\mathbf{x}_j\|_2 \leq n. \tag{53}$$

By definition

$$(\mathbf{A} + \varepsilon\mathbf{E})\mathbf{x}_i(\varepsilon) = \lambda_i(\varepsilon)\mathbf{x}_i(\varepsilon) \tag{54}$$

and since $\lambda(\varepsilon)$ and all components $\mathbf{x}_i(\varepsilon)$ are represented by series that converge, are equate the terms at equal powers of ε in equation (54). Equating the coefficients with ε and taking into account that

$$\lambda_i(\varepsilon) = \lambda_i + k_1\varepsilon + k_2\varepsilon^2 \dots, \tag{55}$$

taking the first approximation of the series

$$\lambda_i(\varepsilon) = \lambda_i + k_1\varepsilon \tag{56}$$

and since

$$k_1 = \frac{{}_i^T\mathbf{E}\mathbf{x}_i}{\mathbf{y}_i^T\mathbf{x}_i} = \frac{\beta_{11}}{s_1}; \tag{57}$$

from (53)

$$|k_i| = \frac{n}{|\mathbf{s}_i|}. \tag{58}$$

That is, for a sufficiently small ε, the leading term λ_i equals $k_1\varepsilon$ and the sensitivity of this eigenvalue first of all depends on $\mathbf{s}_i$.

Conclusions and suggestions

In this work the sensitivity and stability of models of dynamic systems was investigated.

The system and its corresponding model are stable if the roots of the characteristic equation of the perturbed matrix **A** – (matrix $\mathbf{A} + \varepsilon\, \mathbf{E}$) lie in the left half-plane of the complex plane (to the left of the imaginary axis – the ordinate axis). At the same time, the centers of Hershhorin's circles are the diagonal elements of the matrix $(\mathbf{A} + \varepsilon\,\mathbf{E})$, and the radii are determined according to equation (48) for the matrix $(\mathbf{A} + \varepsilon\,\mathbf{E})$.

There are developed numerical methods for calculating the eigenvalues of matrices $\mathbf{A}$ and $(\mathbf{A} + \varepsilon\,\mathbf{E})$: the rotation method for symmetric matrices, the A.M. Danilevsky method with the transformation of the original matrix into the Frobenius matrix, the O.N. Krylov method [38] based on Hamilton-Kelle's identity, and Leverett-Faddeev's method based on Newton's formulas for the sum of the degrees of the roots of the matrix characteristic equation. The last three methods are approximately the same from the point of view of computational costs and do not require symmetric matrix conditions.

The stability of the dynamic system model is determined by the location of the eigenvalues on the complex plane.

The sensitivity of the model to perturbations of the model parameters can be approximately estimated by the first term in the expansion of $\lambda(\varepsilon)$.

An improved methodology for finding the optimal system estimation by using projection methods, as opposed to other methods that use variational computing. Projection methods of investigation allow simultaneous and independent solution of the task of estimation of the dynamic system state vectors and finding optimal control sequences.

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The authors are responsible for the content of the submitted materials in the monograph.

The collective monograph is the result of the generalization of the conceptual work of scientists who consider current topics from such fields of knowledge as: management, management, technical sciences, law, ecology, information sciences and psychological sciences through the prism of international security studies. Content-functional lines and the key direction of the study of psycho- and sociogenesis of personality in age and pedagogical dimensions through the prism of revitalization are highlighted by each researcher in the context of the implementation of an individual sub-theme.

For scientists, educational staff, PhD candidates, masters of educational institutions, university faculties, stakeholders, managers and employees of management bodies at various hierarchical levels, and for everyone, who is interested in current problems of management, technical sciences, law, ecology, information sciences and psychological sciences through the prism of international security studies.

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