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SECTION 16.
SYSTEM ANALYSIS, MODELING AND OPTIMIZATION

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USE OF THE METHOD OF STEEPEST DESCENT TO FIND THE OPTIMUM VALUES OF FUNCTIONS

The conjugate gradient method is an iterative method. A common property of most iterative algorithms is the rapid decrease in the rate of minimization when approaching the minimum point of the functional. Therefore, an important characteristic of iterative algorithms is the actual minimum level of values of the residual functional, to which it is possible to bring the minimization process in real time [1].

To consider the steepest descent method, it's introduced some notation that will be used. The scalar product of two vectors $\boldsymbol{x}^T\boldsymbol{y}$ is the sum of scalars $\sum_{i=0}^{n}\boldsymbol{x}_i\boldsymbol{y}_i$, where $\boldsymbol{x}^T\boldsymbol{y}=\boldsymbol{y}^T\boldsymbol{x}$. If **x** and **y** are orthogonal, then $\boldsymbol{x}^T\boldsymbol{y}=0$. Expressions that transform in a matrix 1×1, such as $\boldsymbol{x}^T\boldsymbol{y}$ and $\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x}$, are treated as scalars.

Initially, the steepest descent method was developed to solve systems of linear algebraic equations of the form [2, 3]:

$$\begin{aligned} &a_{0,0}x_0 + a_{0,1}x_1 + \cdots + a_{0,n-1}x_{n-1} = b_0 \\ &a_{1,0}x_0 + a_{1,1}x_1 + \cdots + a_{1,n-1}x_{n-1} = b_1 \\ &\ldots \\ &a_{n-1,0}x_0 + a_{n-1,1}x_1 + \cdots + a_{n-1,n-1}x_{n-1} = b_{n-1} \end{aligned} \qquad (1)$$

In matrix form *(1)* looks like:

$$\boldsymbol{A}\boldsymbol{x}=\boldsymbol{b}, \qquad (2)$$

where:
$\boldsymbol{x}$ – *unknown vector;*
$\boldsymbol{b}$ – *known vector;*
$\boldsymbol{A}$ – *known square symmetric positive-definite matrix.*

Solving this system is equivalent to finding the minimum of the corresponding quadratic form:

$$f(\boldsymbol{x})=\frac{1}{2}\boldsymbol{x}^T\boldsymbol{A}\boldsymbol{x}-\boldsymbol{b}^T\boldsymbol{x}+\boldsymbol{c}. \qquad (3)$$

The presence of such a connection between the linear transformation matrix **A** and the scalar function *f*(**x**) makes it possible to demonstrate some linear algebra functions with figures (Fig. 1).

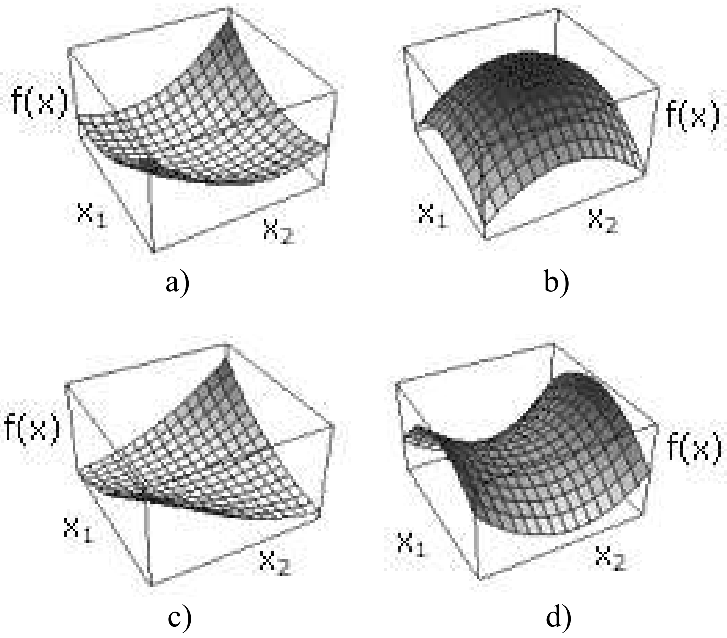

Fig. 1. **Quadratic forms for a positive-definite matrix (a), negative-definite matrix (b), positive-indefinite matrix (c), indefinite matrix (d) [4]**

A matrix **A** is positive definite if for any non-zero vector **x** the expression is true:

$$\boldsymbol{x}^T \boldsymbol{A} \boldsymbol{x} > 0. \qquad (4)$$

To find a positive definite matrix **A**, it is necessary to find the minimum of its quadratic function. Moreover, using the steepest descent method, the minimum of a quadratic function can be found in n steps or less, where n – is the dimension of the unknown vector **x**. Based on the fact that any smooth function in the vicinity of its minimum point is well approximated by a quadratic function, the same method can be used to minimize non-quadratic functions as well. In this case, the method ceases to be finite, but becomes iterative [4].

To begin with, let's consider the method of steepest descent as the usual way to find the extremum of a function. Let's present the algorithm of this method [3]:

Step 1. At the starting point $x(0)$, the gradient is calculated. The movement is carried out in the direction of the antigradient until the objective function decreases.

Step 2. At the point where the function stops decreasing, the gradient is calculated again and the descent continues in the new direction.

Step 3. The process is repeated until the point reaches the minimum.

Figure 2 shows the trajectory of movement to the minimum point by the steepest descent method.

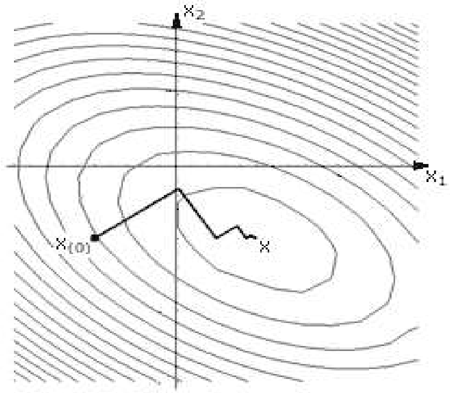

Fig. 2. **The trajectory of movement to the minimum point by the steepest descent method** [4]

Conclusions. In the case of the steepest descent method, each new direction of motion is orthogonal to the previous one.

There is another way to choose a new direction of movement - the method of conjugate directions, which includes the method of conjugate gradients. The conjugate gradient method is a further development of the steepest descent method, which combines two concepts: the gradient of the objective function and the combined direction of vectors, which will be considered below.

References:

1. Тихонов А.Н. & Гончаровский А.В. & Степанов В.В. & Ягола А.Г. (1983). *Регуляризирующие алгоритмы и априорная информация*. Москва: Наука.
2. Решение СЛУ методом сопряженных градиентов. (2012). Removed from http://www.hpcc.unn.ru/?dir=847.
3. Димова Г.О. (2021). Знаходження оптимальних значень функцій із застосуванням методу спряжених градієнтів. *Таврійський науковий вісник. Серія: Технічні науки. Херсонський державний аграрно-економічний університет.* (с. 3-9). Вип. 3. Херсон: Видавничий дім «Гельветика».
4. Некипелов Н. Метод сопряженных градиентов — математический аппарат. Removed from https://basegroup.ru/community/articles/conjugate.

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