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Equilibrium stability of a ribbed three-layer shell

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ABSTRACT

A computational model is constructed and an algorithm for investigation of the stability of a three-layered sloping shell supported by transverse stiffness ribs is developed. The variational method, based on the principle of possible displacements, is used to derive the differential stability equations for the region of the shell enclosed between the edges, as well as the conditions along the edge lines and along the edges of the shell. There was developed a program for the numerical implementation of the author's methodology, it was implemented in the Wolfram Mathematica environment. It is shown that there is a finite value of the moment of inertia of the ribs that supports the shell, at which the maximum critical stress (the critical moment of inertia of the rib) can be reached, which is determined from the stability equation. As an example, we consider a square in plan shell, supported by one and three stiffness ribs. The values of the critical moment of inertia of the rib are presented, which were determined both with regard to the edge Reissner effect and without taking it into account. The dependences of the critical load parameter on the linear dimensions of the shell, reinforced by one and three transverse stiffness ribs, are plotted.

1. INTRODUCTION

Man-caused emergencies arise due to the depletion of the resource of buildings and structures [1], [2]. Most of the damage occurred due to unsatisfactory technical condition of building structures [3]. The use of concrete as a building material increases resistance to aggressive environments and increases durability [4]. Creation of new construction materials with high strength characteristics [5], low weight and low thermal conductivity has led to the appearance of a variety of enclosing structures in the form of plates, slabs and shells consisting of two or more layers [6– 8]. The most effective were three-layered structures, consisting of a composition of two sufficiently strong outer layers of a small thickness and a light inner layer, designed primarily to provide high thermal and sound insulation characteristics [9– 11]. One of the significant problems arising in the construction of three-layer structures is the loss of stability at relatively low loads. For its solution it is used the arrangement of stiffness ribs, which supports the structure in one or two directions. Such a solution leads to an increase in strength and deformation characteristics, an increase in the range of critical forces of buckling with a slight increase in the weight of the entire structure. However, the methods for calculating of structures supported by stiffness ribs are significantly more complicated, and development of new approaches remains urgent [12], [13].

2. Materials and methods

The variational method, based on the principle of possible displacements, is used to derive the differential stability equations for the region of the shell enclosed between the ribs, as well as the conditions along the ribs lines and along the edges of the shell. For the numerical realization of authors method there was developed program "Three-layered shell – II". It is implemented in the Wolfram Mathematica 11 [14].

3. Problem formulation

It is considered the stability of three-layered sloping shell with light transversally isotropic aggregate, which is supported by transverse ribs of equal stiffness and are located on equal distances of each other (Fig. 1).

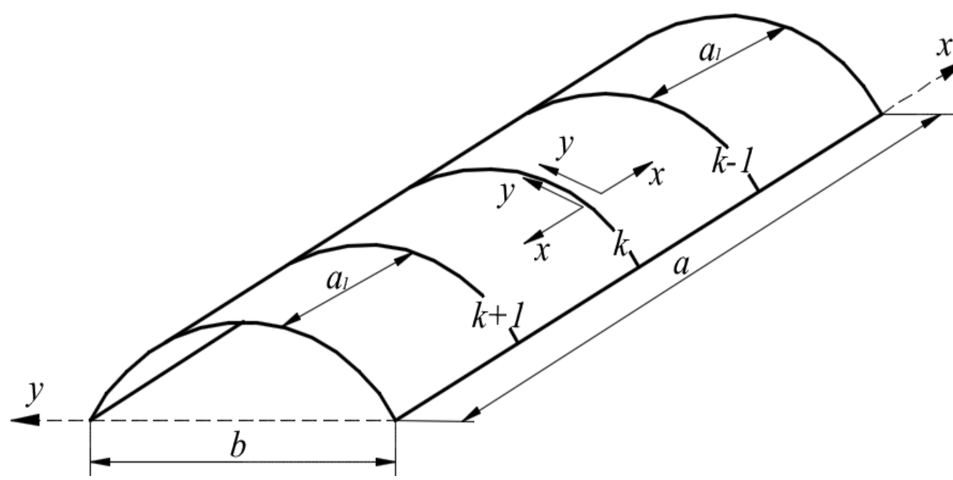

Figure. 1. Scheme of three-layered sloping shell, supported by transverse stiffness ribs.

4. Research results

The differential equations of stability of a section of the shell closed between the ribs and also the conditions along the lines of the ribs and along the edges of a three-layered sloping shell, which is supported by transverse stiffness ribs, with the free support of the edges, are obtained by a variational method using the Ostrogradskii-Hamilton functional [15]:

$$\nabla^4\Phi + \frac{\overline{B}}{R}\frac{\partial^2}{\partial x^2}\left(\varphi - \frac{Bh}{G_3}\nabla^2\varphi\right) = 0\,; \qquad (1)$$

$$\nabla^4\varphi - \frac{1}{RD^*}\frac{\partial^2\Phi}{\partial x^2} + \frac{2\mathrm{T}_1}{D^*}\frac{\partial^2}{\partial x^2}\left(1 - \frac{Bh}{G_3}\nabla^2\right)\varphi = 0\,; \qquad (2)$$

$$\Psi - \frac{1-\mu}{2G_3}Bh\nabla^2\Psi = 0\,. \qquad (3)$$

The system of differential equations has been transformed by the displacement function F [16]:

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$$\nabla^4\nabla^4 F + \frac{\overline{B}}{R^2 D^*}\frac{\partial^4}{\partial x^4}\left(1-\frac{Bh}{G^3}\nabla^2\right)F + \frac{2T_1}{D^*}\frac{\partial^2}{\partial x^2}\left(1-\frac{Bh}{G^3}\nabla^2\right)\nabla^4 F = 0 \quad (4)$$

The solution of equations (3) and (4) for the section of the shell closed between the ribs is sought in the form

$$F = f_1(x)\sin\frac{\pi}{b}y; \quad (5)$$

$$\Psi = f_2(x)\cos\frac{\pi}{b}y. \quad (6)$$

After substituting (5) and (6) in (3) and (4), the solution of the equations takes the form

$$F = \sin\frac{\pi y}{b}((\cos(x\varphi_1)(C_1+C_3)+\sin(x\varphi_1)(C_2-C_4))\rho_1^x + (\cos(x\varphi_2)(C_5+C_7)+$$
$$+\sin(x\varphi_2)(C_6-C_8))\rho_2^x); \quad (7)$$

$$\psi = (C_9\cos(\beta x)+C_{10}\sin(\beta x))\cos\frac{\pi y}{b}. \quad (8)$$

Here $tg\varphi_1 = r/s$; $tg\varphi_2 = d/c$; $\rho_1 = \left|\sqrt{s^2+r^2}\right|$; $\rho_2 = \left|\sqrt{c^2+d^2}\right|$; s, c – real, r, d – complex roots of the characteristic equation

$$b^8(1-k_0 m_t)\lambda^8 + b^6\pi^2(m_t - \alpha^2 k_0)\lambda^6 + b^4\pi^4\alpha^2(1+k_0)\lambda^4 + b^2\pi^6 m_t(1+k_0)\lambda^2 + \pi^8 = 0,$$

where $k_0 = \frac{\pi^2 Bh}{G_з b^2}$; $\alpha^2 = \frac{\overline{B}b^4}{R^2 D^*\pi^4}$; $\beta = \pi\sqrt{\frac{2}{b^2(1-\mu)k_0} - \frac{1}{b}}$ – complex root of characteristic equation

$$\left[(1-\mu)t_0\frac{b^3}{\pi^2}\right]f_2''(x) + \left[2b - t_0 b(1-\mu)\right]f_2(x) = 0.$$

Assuming that there are diaphragms on the edges of the shell, the boundary conditions for the case of free support will be written in the form:

$$at\ x = 0, a \quad w = \frac{\partial u_\beta}{\partial x} = v_\beta = v_\alpha = \frac{\partial^2\Phi}{\partial y^2} = 0; \quad (9)$$

$$at\ y = 0, b \quad w = \frac{\partial v_\beta}{\partial y} = u_\beta = u_\alpha = \frac{\partial^2\Phi}{\partial x^2}. \quad (10)$$

Assuming for each section its own coordinate axes, we arrange them at the beginning of the section and denote the values $f_1(x)$ at the start and end of section as η_k and η_{k+1}, values f_1'' – as μ_k and μ_{k+1}, values f_1^{IV} – as ς_k and ς_{k+1}, values f_1^{VI} – as ξ_k and ξ_{k+1}, values f_2'' – as φ_k and ϕ_{k+1}.

Using these conditions, we express through them the values of arbitrary constants of solutions (7) and (8), which are determined from the system of equations:

$$\eta_k = C_1^k + C_3^k + C_5^k + C_7^k;$$

$$\mu_k = (C_1^k + C_3^k)(p_5^2 - \varphi_1^2) + (C_5^k + C_7^k)(p_6^2 - \varphi_2^2) + 2(C_2^k - C_4^k)p_5\varphi_1 + 2(C_6^k - C_8^k)p_6\varphi_2; \quad (11)$$

$$\zeta_k = (C_1^k + C_3^k)(p_5^4 - 6p_5^2\varphi_1^2 + \varphi_1^4) + (C_5^k + C_7^k)(p_6^4 - 6p_6^2 + \varphi_2^4) +$$
$$+(C_2^k - C_4^k)(4p_5^3\varphi_1 - 4p_5\varphi_1^3) + (C_6^k - C_8^k)(4p_6^3\varphi_2 - 4p_6\varphi_2^3);$$
$$\xi_k = (C_1^k + C_3^k)(p_5^6 - 15p_5^4\varphi_1^2 + 15p_5^2\varphi_1^4 - \varphi_1^6) + (C_5^k + C_7^k)(p_6^6 - 15p_6^4\varphi_2^2 + 15p_6^2\varphi_2^4 - \varphi_2^6) +$$
$$+(C_2^k - C_4^k)(6p_5^5\varphi_1 - 20p_5^3\varphi_1^3 + 6p_5\varphi_1^5) + (C_6^k - C_8^k)(6p_6^5\phi_2 - 20p_6^3\phi_2^3 + 6p_6\phi_2^5);$$
$$\varphi_k = \beta^2 C_{10}^k;$$
$$\eta_{k+1} = ((C_1^{k+1} + C_3^{k+1})p_1 + (C_2^{k+1} - C_4^{k+1})p_2)\rho_1^{a_1} + ((C_5^{k+1} + C_7^{k+1})p_3 + (C_6^{k+1} - C_8^{k+1})p_4)\rho_2^{a_1};$$

$$\eta_{k+1} = ((C_1^{k+1} + C_3^{k+1})p_1 + (C_2^{k+1} - C_4^{k+1})p_2)\rho_1^{a_1} + ((C_5^{k+1} + C_7^{k+1})p_3 + (C_6^{k+1} - C_8^{k+1})p_4)\rho_2^{a_1};$$
$$\mu_{k+1} = (C_1^{k+1} + C_3^{k=1})(\rho_1^{a_1}(p_1(p_5^2 - \phi_1^2) - 2p_2p_5\varphi_1)) + (C_2^{k+1} - C_4^{k+1})(\rho_1^{a_1}(p_2(p_5^2 + \varphi_1^2) + 2p_1p_5\varphi_1)) +$$
$$+(C_5^{k+1} + C_7^{k+1})(\rho_2^{a_1}(p_3(p_6^2 - \varphi_2^2) - 2p_4p_6\varphi_2)) + (C_6^{k+1} + C_8^{k+1})(\rho_2^{a_1}(p_4(p_6^2 - \varphi_2^2) + 2p_3p_6\varphi_2));$$
$$\zeta_{k+1} = (C_1^{k+1} + C_3^{k+1})(\rho_1^{a_1}(p_1(p_5^4 - 6p_5^2\varphi_1^2 + \varphi_1^4) - 4p_2p_5\phi_1(p_5^2 + \varphi_1^2))) +$$
$$+(C_2^{k+1} - C_4^{k+1})(\rho_1^{a_1}(p_2(p_5^4 + 6p_5^2\varphi_1^2 + \varphi_1^4) + 4p_1p_5\varphi_1(p_5^2 - \varphi_1^2))) +$$
$$+(C_5^{k+1} + C_7^{k+1})(\rho_2^{a_1}(p_3(p_6^4 - 6p_6^2\varphi_2^2 + \varphi_2^4) - 4p_4p_6\varphi_2(p_6^2 + \varphi_2^2))) +$$
$$+(C_6^{k+1} + C_8^{k+1})(\rho_2^{a_1}(p_4(p_6^4 + 6p_6^2\varphi_2^2 + \varphi_2^4) + 4p_3p_6\varphi_2(p_6^2 - \varphi_2^2)));$$
$$\xi_{k+1} = (C_1^{k+1} + C_3^{k+1})(\rho_1^{a_1}(p_2p_5(p_5^5 - 6p_5^4\varphi_1 - 20p_5^2\varphi_1^3 - 6\varphi_1^5) + p_2(p_5^6 - 15p_5^4\varphi_1^2 + 15p_5^2\varphi_1^4 - \varphi_1^6))) +$$
$$+(C_2^{k+1} - C_4^{k+1})(\rho_1^{a_1}\varphi_1(p_1\varphi_1(15p_5^4 + 15p_5^2\varphi_1^2 - \varphi_1^4) + p_1(6p_5^5 - 20p_5^3\varphi_1^2 + 6p_5\varphi_1^4))) +$$
$$+(C_5^{k+1} + C_7^{k+1})(\rho_2^{a_1}(p_4p_6(C_5^{k+1} + C_7^{k+1}) + p_4(p_6^6 - 15p_6^4\varphi_2^2 + 15p_6^2\varphi_2^4 - \varphi_2^6))) +$$
$$+(C_6^{k+1} + C_8^{k+1})(\rho_2^{a_1}\varphi_2(p_3\varphi_2(15p_6^4 + 15p_6^2\varphi_2^2 - \varphi_2^4) + p_3(6p_6^5 - 20p_6^3\varphi_2^2 + 6p_6\varphi_2^4)));$$
$$\varphi_{k+1} = -\beta^2 C_9^{k+1}\cos(\beta a_1) - \beta^2 C_{10}^{k+1}\sin(\beta a_1).$$

There is denoted in equation (11):

$$\cos(a_1\varphi_1) = p_1;\ \cos(a_1\varphi_2) = p_3;\ \sin(a_1\varphi_1) = p_2;\ \sin(a_1\varphi_2) = p_4;\ \log\rho_1 = p_5;\ \log\rho_2 = p_6.$$

When we consider the (k-1)-th section, we take the origin at its end and direct the x-axis in the opposite direction. Then for it $f_1(x)$ and $f_2(x)$ will have the same form as (7) and (8), and arbitrary constants (we denote them as C_1^{k-1}) will be determined from (11), if we replace in them $\eta_{k+1}, \mu_{k+1}, \zeta_{k+1}, \xi_{k+1}, \varphi_{k+1}$ with $\eta_{k-1}, \mu_{k-1}, \zeta_{k-1}, \xi_{k-1}, \varphi_{k-1}$.

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The conditions along the line of the k-th rib, taking into account the different directions of the x-axes for contiguous sections, which are obtained from the variational equation [6], we write in the form

$$\frac{\partial^4}{\partial x^3 \partial y}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)_{x=+0}+\frac{\partial^4}{\partial x^3 \partial y}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)_{x=-0}=$$

$$=\frac{B_{pk}}{\overline{B}}\left[\frac{\partial^5}{\partial x^4 \partial y}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)-\mu\frac{\partial^5}{\partial x^2 \partial y^3}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)\right]_{x=0};$$

$$\int\left[\frac{\partial^4}{\partial x^2 \partial y^2}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)-\mu\frac{\partial^4}{\partial x^4}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)\right]_{x=+0}dx=$$

$$=-\int\left[\frac{\partial^4}{\partial x^2 \partial y^2}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)-\mu\frac{\partial^4}{\partial x^4}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)\right]_{x=-0};$$

$$\left(\frac{\partial}{\partial x}\nabla^4 F+\frac{\partial \psi}{\partial y}\right)_{x=+0}=-\left(\frac{\partial}{\partial x}\nabla^4 F+\frac{\partial \psi}{\partial y}\right)_{x=-0};$$

$$\left(\frac{\partial}{\partial x}\nabla^6 F\right)_{x=+0}+\left(\frac{\partial}{\partial x}\nabla^6 F\right)_{x=-0}=-\left\{\frac{D_{pk}}{D^*}\left(1-\frac{Bh}{G_з}\nabla^2\right)\frac{\partial^4}{\partial y^4}\nabla^4 F+\right.$$

$$\left.+\frac{B_{pk}}{\overline{B}}\frac{\overline{B}}{D^* R^2}\left[\frac{\partial^4}{\partial x^4}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)-\mu\frac{\partial^4}{\partial x^2 \partial y^2}\left(F-\frac{Bh}{G_з}\nabla^2 F\right)\right]-\left(1-\frac{Bh}{G_з}\nabla^2\right)\frac{P_p}{D^*}\nabla^4 F\right\}_{x=0}; \quad (12)$$

$$\left[-\frac{\partial \psi}{\partial y}+\frac{Bh}{G_з}\frac{\partial}{\partial y}\nabla^6 F\right]_{x=+0}=0$$

Substituting solutions of equations (6) and (7) into (12), we have a system of equations where the values of arbitrary constants C_i^{k-1} and C_i^{k+1} are determined by expressions (11).

The solution of this system is sought in the form

$$\eta_k = A_* \sin\frac{k\pi}{m};\ \mu_k = B_* \sin\frac{k\pi}{m};\ \zeta_k = C_* \sin\frac{k\pi}{m};\ \xi_k = D_* \sin\frac{k\pi}{m};\ \varphi_k = E_* \sin\frac{k\pi}{m}. \quad (13)$$

Unknown $\eta_k, \mu_k, \zeta_k, \xi_k, \phi_k$, which enter into this system, must satisfy the condition of periodicity of solutions:

$$\eta_0 = \mu_0 = \zeta_0 = \xi_0 = \varphi_0 = \eta_m = \mu_m = \zeta_m = \xi_m = \varphi_m = 0. \quad (14)$$

Equating the determinant composed of the coefficients $A_*, B_*, C_*, D_*, E_*,$ to zero, we obtain the stability equation for determining the stiffness parameter of a three-layered sloping shell, which is supported by regular transverse stiffness ribs, with the hinged support of the edges. If you set the critical stress (m_t), you can determine the corresponding value of γ, and according to it, according to $\gamma = D_p / bD^*$, found the necessary moment of inertia of the supporting rib. When determining the maximum value of γ, the critical stress (m_t) should not exceed the critical stress of the shell section between undeformed ribs.

In the problem, there is a finite value of the moment of inertia of the shell-supporting ribs, at which the above-mentioned maximum critical stress (the critical moment of inertia of the rib γ_0), which is also found from the stability equation, can be achieved. As an example, we consider a square in plan shell, supported by one and three ribs of rigidity.

In the tables 1 and 2 there are shown values γ_0, which were determined both taking into account the edge Reissner effect, and without taking it into account.

Table 1. Shell, supported by one rib

	Shell, supported by one rib, at $a/b=1$ and $\alpha^2=5$				Shell, supported by one rib, at $a/b=1$ and $\alpha^2=10$			
k_0	0.05	0.1	0.2	0.3	0.05	0.1	0.2	0.3
γ_0	0.230	0.196	0.167	–	0.502	0.469	0.402	–
γ_0^*	0.221	0.185	0.138	–	0.488	0.456	0.381	–

Table 2. Shell, supported by three ribs

	Shell, supported by three ribs, at $a/b=1$ and $\alpha^2=5$				Shell, supported by three ribs, at $a/b=1$ and $\alpha^2=10$			
k_0	0.05	0.1	0.2	0.3	0.05	0.1	0.2	0.3
γ_0	0.696	0.438	0.366	–	1.166	0.937	0.516	–
γ_0^*	0.679	0.430	0.338	–	1.098	0.881	0.360	–

5. Conclusions

As can be seen from the calculation results, at $k_0 \geq 0,2$ the edge effect of Reissner has a significant effect on the value $\gamma = \gamma_0$. With an increase in the number of ribs, this effect decreases.

The graphs (Fig. 2, Fig. 3) show the values of the parameter m_t, which were also determined by taking into account the edge Reissner effect (solid lines on the graphs) and without taking it into account. As can be seen from the graphs, at values $\gamma < \gamma_0$ and $k_0 < 0,4$ the edge Reissner effect has a negligible effect on the value of m_t.

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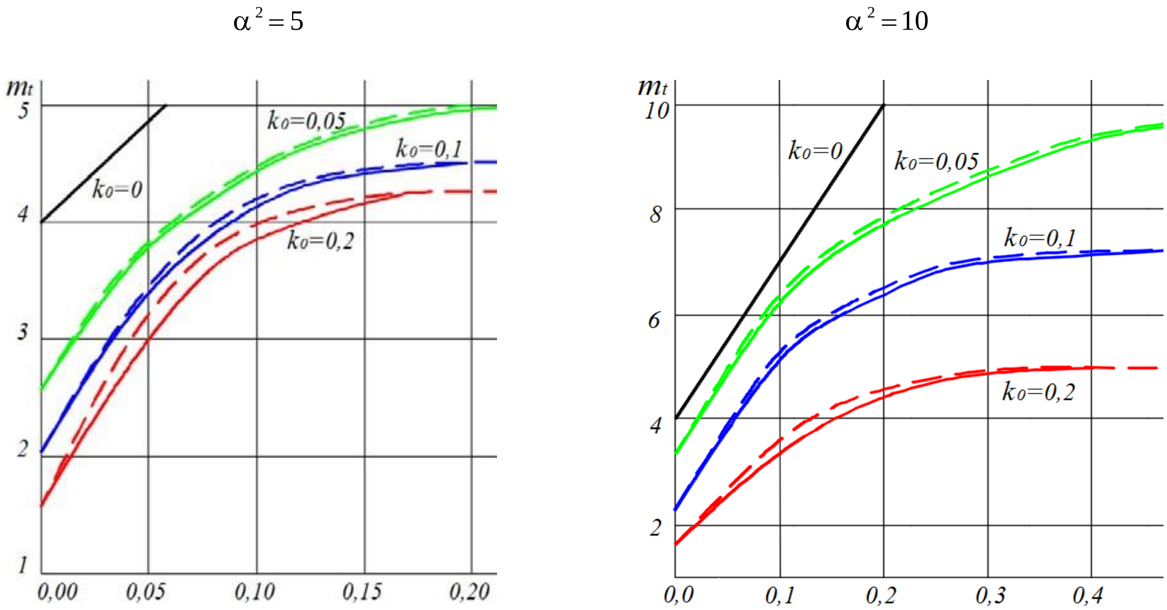

Figure 2. Dependence of the critical load parameter m_t on the linear dimensions of the shell, supported by a single transverse stiffness rib.

At values $k_0 < 0,4$, support of the shell with stiffener ribs results in only a slight increase in the critical load, because in this case the shell buckling with the formation of a large number of half-waves in the longitudinal direction.

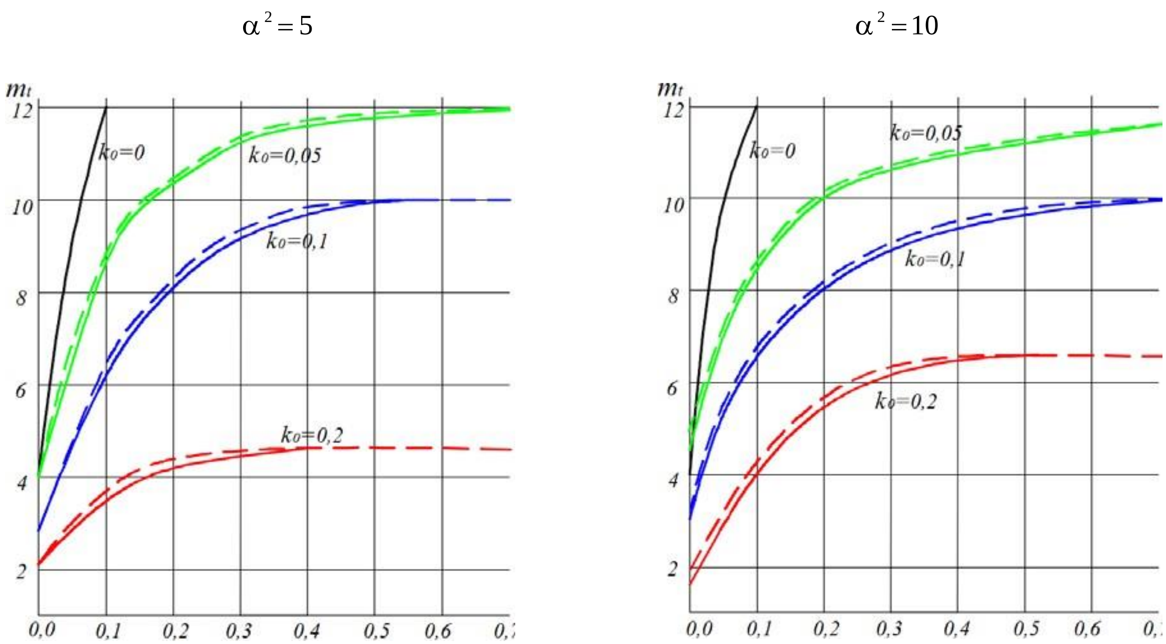

Figure 3. Dependence of the critical load parameter mt on the linear dimensions of the shell, supported by three transverse stiffness ribs.

At values of the curvature parameter of the shell $\alpha^2 > 5$, an increase in the number of stiffeners does not lead to an increase in the critical load.

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