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# Mathematical model of the vehicle initial rectilinear motion during moving uphill

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**Abstract.** In the article the mathematical model of the vehicle motion with elastic tires, which describes the phases of the initial rectilinear motion on the longitudinal slope of the road is shown. The mathematical model is valid for vehicle with two axles, one of which is driving axle and provided that the stopped vehicle on a slope is kept from rolling only by braking the non-driving axle. The aim of the study is to determine the equations to describe the process of vehicle initial rectilinear motion during moving uphill, which take into account the tires deformation energy on the stationary vehicle braking wheels. The presented equations described in the article are reflect the dynamic state of the stationary vehicle on a slope and the phase of its initial moving off uphill after brining the engine torque through the transmission to the wheels of the drive axle. The system of equations describing the vehicle dynamic behavior after brining torque to the drive wheels takes into account their false slip. The system of equations is obtained from the general dynamic equation in the form of balance of the elementary work of external forces and moments with the absence of external slipping of the driving wheels. The mathematical model of the vehicle initial rectilinear motion on a longitudinal slope of a highway described in the article can be used in theoretical basis for analyzing and synthesizing control algorithms for automatic driver assistance systems during vehicle initial motion.

## 1. Introduction

Driving vehicle uphill with frequent stops and subsequent resumption of movement, especially in dense traffic conditions, requires a high driver qualification. The quality of process control has a significant impact on the traffic flow dynamics, and traffic safety. To reduce the driver's workload and increase traffic safety, automatic systems are included in the design of vehicles to assist in driving. The most promising for improvement are electronically controlled systems endowed with the properties of adaptation to external and internal disturbing factors. In this case, increases comfort and control accuracy of the vehicle under the action of internal and external disturbing factors. The design and creation of any complex technical device and, in particular, automatic systems of driver assistance in driving a vehicle implies their theoretical studies. At the same time, the volume and quality of theoretical research results directly depend on the correspondence of the mathematical models of the control system and the control object to the goals and objectives of the research.



## 2. Analysis of publications

The versatile researches of the vehicles moving off process in the mode of friction clutch turn on and slipping, were made by authors [1]. The calculation and analysis processes of the moving off, acceleration and movement of a tractor with a trailer was made in [2]. A number of studies [3–10] are devoted to the modeling and research of clutch operations and their automatic control systems when the vehicle moves off. Much attention is paid to the study of the vehicle dynamics during acceleration and deceleration, as well as its dependence on control parameters [11–14]. Modeling of the vehicle initial movement dynamics, the working processes of the control systems and their algorithms are the subject of research [15–23]. All mentioned studies are based on the mathematical description of the dynamic state of the vehicle. It should be noted that vehicle initial movement uphill differs in that at the beginning of the movement the braking wheels are in stress-strain state. Obviously, this circumstance has scientific and practical interest for the creation of automatic control systems to help the driver during the vehicle initial movement mode.

## 3. The purpose and objectives of the study

The aim of the study is to develop the mathematical model of the vehicle initial rectilinear uphill motion taking into account the tires deformation energy of the braking wheels.

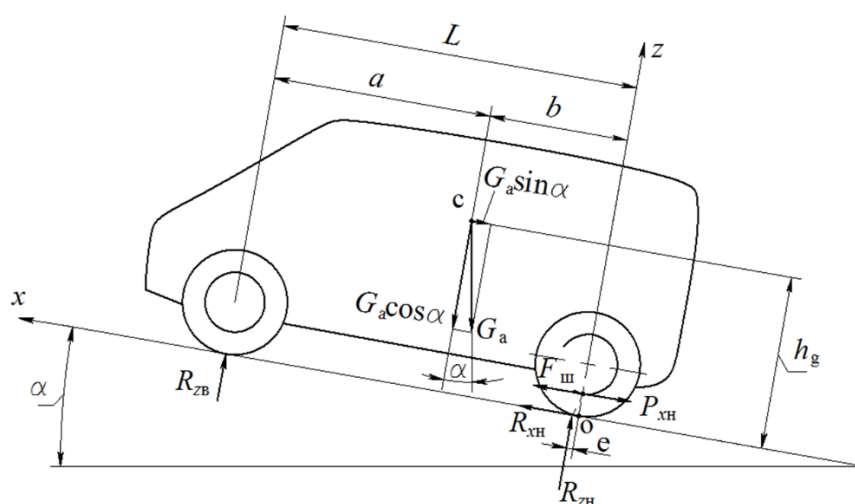
The mathematical model of the vehicle initial rectilinear uphill motion should reflect the phases of its movement with braked driven wheels:

- phase of movement with non-rotating wheels in the zone of their elastic deformation;
- phase of movement with rotating wheels.

Taking into account the tires deformation energy of the braked wheels with various dynamic conditions of the vehicle initial uphill motion, it is possible to clarify the requirements for the algorithm of automatic vehicle control assistance systems and, as a consequence, to improve the quality of control process.

## 4. Description of the mathematical model of the vehicle rectilinear motion

To obtain the mathematical model of the of the vehicle rectilinear uphill motion, let us analyze the loads acting to the stationary vehicle on a slope. The scheme of external influences on a stationary vehicle is shown in figure 1.



**Figure 1.** Scheme of external influences acting on a stationary vehicle:  $xoz$  – fixed coordinate system;  $L$  – vehicle base;  $a$ ,  $b$ ,  $h_g$  – coordinates of the vehicle center of mass.

In drawing up the scheme of loads acting on a stationary vehicle, the following assumptions and conditions are adopted:

- biaxial vehicle with front drive wheels;
- only the wheels of the non-driving rear axle are braked;
- braking wheels are blocked;
- wheels of the same axles have the same loads, size and characteristics;
- there is no air movement;
- all wheel suspension is absolutely tough;
- the surface on the slope of the road is absolutely flat and firm, located at an angle  $\alpha$  to the horizon.

The system of external loads under the accepted assumptions and conditions are:  $G_a = m_a \cdot g$  – vehicle gravity ( $m_a$  – vehicle mass;  $g$  – acceleration of gravity);  $R_{zw}$ ,  $R_{zh}$  – normal road reactions to driving and non-driving wheels;  $R_{zh}$  – longitudinal reaction of the road to the non-driving braking wheels.

The equation of the vehicle dynamic state in this case is described by the equilibrium equation:

$$R_{zh} - G_a \cdot \sin \alpha = 0, \quad (1)$$

where  $G_a \cdot \sin \alpha = P_\alpha$  is the uphill motion resistance force.

The uphill motion resistance force  $P_\alpha$  in case of stationary vehicle on a slope is inherently a rolling force. This force, in accordance with the condition of stationary vehicle, is compensated by a longitudinal tangential reaction of the road to the braking wheels  $R_{zh}$ . In this case, the equality is true:

$$R_{zh} = P_{\phi h} \quad (2)$$

where  $P_{\phi h}$  is the actual adhesion force of the driven braking wheels with road surface.

The stationary condition of the vehicle on the uphill and equation (2) are provided with equation:

$$\frac{M_{th}}{r_{dh}} \geq G_a \cdot \sin \alpha, \quad (3)$$

where  $M_{th}$  is the braking moment to the driven wheels, created by the braking mechanisms;  $r_{dh}$  is the dynamic radius of the wheels.

The rolling force  $P_\alpha$  causes longitudinal deformation of the tires of the braking wheels. As a result, the normal road reaction to the braking wheels acquires a longitudinal displacement by the value "e". The stationary condition of the vehicle corresponds to the equality of forces and reactions:

$$P_\alpha = R_{zh} = P_{\phi h} = F_t, \quad (4)$$

where  $F_t = c_t \cdot e$  is the force of the tire elastic deformation in the longitudinal direction ( $c_t$  is the tires longitudinal stiffness coefficient of the braked axis).

Normal reactions to the axes of the stationary vehicle on a slope according to figure 1 determine the equations:

$$\left. \begin{aligned} R_{zh} &= G_a \cdot \cos \alpha \frac{a + \operatorname{tg} \alpha \cdot h_g}{L - e}, \\ R_{zb} &= G_a \cdot \cos \alpha \frac{b - e - \operatorname{tg} \alpha \cdot h_g}{L - e}. \end{aligned} \right\} \quad (5)$$

The forward movement of the vehicle can begin when the engine power is applied to the drive wheels. The applied energy to the driving wheels is characterized by the torque  $M_k$ , acting on the driving axle. The scheme in figure 2 shows the loads acting to the vehicle when the torque  $M_k$  is applied to the driving wheels.



where  $\delta_{rt}$  is the coefficient of accounting rotating masses of the vehicle;  $m_a$  is the vehicle mass;  $\ddot{x}$  is the longitudinal acceleration of the vehicle.

The coefficient  $\delta_{rt}$  of accounting rotating mass of the vehicle characterizes the increase of the vehicle inertia when the angular acceleration of the rotating engine masses, transmission and wheels:

$$\delta_{rt} = 1 + \frac{J_e \cdot u_{rt}^2 \cdot \eta_{rt} + \sum J_{kv}}{r_{kv} \cdot r_{dr} \cdot G_a} \cdot g, \quad (9)$$

where  $J_e$  is the inertial moment of the rotating engine masses;  $\sum J_{kv}$  is the total inertial moment of the driving wheels, axles and differential;  $r_{kv}$  is the radius of the drive wheels;  $r_{dr}$  is the dynamic radius of the drive wheels.

The moment of rolling resistance of the drive wheels in equation (6) is determined by the equation:

$$M_{fw} = P_{fw} \cdot r_d, \quad (10)$$

where  $P_{fw}$  is the rolling resistance force of the drive wheels.

The rolling resistance force  $P_{fw}$  of the drive wheels on a solid surface is caused by hysteresis losses in the tire during its deformation and kinematic losses in the patch of its contact with the surface and is determined by the equation:

$$P_{fw} = f_{kw} \cdot R_{zw}, \quad (11)$$

where  $f_{kw}$  is the coefficient of rolling resistance of the drive wheels.

To determine the coefficient of rolling resistance  $f_{kw}$  of the driving wheels, let us find the elementary work of the rolling resistance forces of the driving wheels  $A_{fw}$  during vehicle initial uphill motion, as:

$$A_{fw} = A_{kw} - A_f, \quad (12)$$

where  $A_{kw}$  is the elementary energy applied to the drive wheel;  $A_f$  is the elementary useful work, made by driving wheel.

Elementary energy applied to the drive wheel  $A_{kw}$  is reflected by the first element in equation (6):

$$A_{kw} = M_k \cdot \delta\varphi_{kv}.$$

Elementary useful work  $A_f$ , made by driving wheel, is an elementary work of the sum of the longitudinal forces  $\sum P_x$  acting to the vehicle, with an elementary movement of the body  $\delta x$ :

$$A_f = \sum P_x \cdot \delta x. \quad (13)$$

The equation that determines the sum of the longitudinal forces acting to the vehicle at the time of the initial uphill motion, has form:

$$\sum P_x = \frac{M_k}{r_{dw}} - \frac{M_{fw0}}{r_{dw}} - G_a \cdot \sin \alpha + F_t, \quad (14)$$

where  $M_{fw0}$  is the moment of rolling resistance of the wheels of the driving axle in the driven mode of their rolling.

Taking into account the detailed description of the components on the right side of equation (12), and also due to the false slip of the driving wheels [14], even in the absence of external skidding,

elementary longitudinal movement of the wheel axis is proportional to the rolling radius of the driving wheel  $\delta x = \delta\varphi_{kv} \cdot r_{kv}$ , the equation (12) will take form:

$$A_{fw} = M_k \cdot \delta\varphi_{kv} - \left( \frac{M_k}{r_{dr}} - \frac{M_{fv0}}{r_{dr}} - G_a \cdot \sin \alpha + F_t \right) \cdot r_{kv} \cdot \delta\varphi_{kv}. \quad (15)$$

In this case, the elementary work of the rolling resistance force is equal to:

$$A_{fw} = P_{fw} \cdot r_{kv} \cdot \delta\varphi_{kv}. \quad (16)$$

Taking into account equation (16), we rewrite equation (15) in the form:

$$P_{fw} \cdot r_{kv} \cdot \delta\varphi_{kv} = M_k \cdot \delta\varphi_{kv} - \left( \frac{M_k}{r_{dr}} - \frac{M_{fv0}}{r_{dr}} - G_a \cdot \sin \alpha + F_t \right) \cdot r_{kv} \cdot \delta\varphi_{kv}. \quad (17)$$

From equation (17) we express the force of the rolling resistance of the driving wheels:

$$P_{fw} = \frac{M_k}{r_{dr}} - \frac{M_k}{r_{dr}} + \frac{M_{fv0}}{r_{dr}} - G_a \cdot \sin \alpha + F_t = M_k \cdot \left( \frac{r_{dr} - r_{kv}}{r_{dr} \cdot r_{kv}} \right) + M_{fv0} \frac{1}{r_{dr}} - G_a \cdot \sin \alpha + F_t. \quad (18)$$

The rolling resistance coefficient is defined as the ratio of the rolling resistance force of the drive wheels (18) and the normal reaction  $R_{zw}$  on this axis:

$$f_{kv} = \frac{M_{fv0}}{R_{zw} \cdot r_{dr}} + \frac{M_k}{R_{zw}} \cdot \left( \frac{r_{dr} - r_{kv}}{r_{dr} \cdot r_{kv}} \right) + \frac{G_a \cdot \sin \alpha - F_t}{R_{zw}}. \quad (19)$$

The rolling resistance moment of the driving wheels  $M_{fv0}$  is due to hysteresis losses in the rolling tire when the normal deformation of its sections in contact with the surface changes. This moment is formed by the normal reaction of the bearing surface of the road on the arm  $a_{fw}$ , which is caused by the longitudinal displacement of the total vector  $R_{zw}$  relative to the axis of the driving wheels when they are rolling in the driven mode [24–31]. Therefore, for the rolling in steady motion mode, it can be written:

$$M_{fv0} = R_{zw} \cdot a_{fw}. \quad (20)$$

If the condition is true:

$$M_{fv0} \geq M_k, \quad (21)$$

equation (19) is converted to:

$$f_{kv} = \frac{M_k}{R_{zw}} \cdot \frac{1}{r_{kv}} + \frac{G_a \cdot \sin \alpha - F_t}{R_{zw}}. \quad (22)$$

From the equation, it follows that the coefficient of rolling resistance of the driving wheels  $f_{kv}$  at the beginning of the uphill motion depends on the elastic force to the braking driven wheels. Obviously, in the case of the vehicle initial uphill motion when the driven wheels are braked, the rolling resistance coefficient of the driving wheels is zero due to the absence of tangential deformation of the tires. After applying the torque from the transmission to the driving wheels of the stationary vehicle, taking into account equation (21), their rolling resistance coefficient increases from zero to the

value determined by equation (22). The subsequent increase of the torque to the driving wheels causes an increase in the rolling resistance coefficient of the driving wheels in accordance with the equation:

$$f_{kv} = \frac{a_{rw}}{r_{dr}} + \frac{M_k}{R_{zw}} \cdot \left( \frac{r_{dr} - r_{kv}}{r_{dr} \cdot r_{kv}} \right) + \frac{G_a \cdot \sin \alpha - F_t}{R_{zw}}. \quad (23)$$

The initial uphill motion of the vehicle is possible when the equation:

$$M_k \geq M_{fv0} + (P_\alpha - F_t) \cdot r_d. \quad (24)$$

If equation (24) is true, the vehicle body has a longitudinal displacement  $x$ . In this case, the elastic force of the braking driven wheels decreases in accordance with the equation:

$$F_t = c_t \cdot (e - x), \quad (25)$$

the rolling resistance coefficient of the drive wheels increases in proportion to the torque in accordance with equation (23), and the possibility of uniform movement of the vehicle body is determined by the equation:

$$M_k > M_{fv0} + [P_\alpha - c_t \cdot (e - x)] \cdot r_{dh}. \quad (26)$$

It should be noted that in this case the vehicle body has a longitudinal movement due to the applying energy of engine capacity to the driving wheels and the energy of the elastic deformation of the braking driven wheels. The equation of the vehicle body uniform movement (26) is true only with an increase of the torque  $M_k$ . This is due to the fact that with an increase of the longitudinal displacement " $x$ ", the value of the elastic force of the braking wheels decreases and the rolling resistance of the driving wheels  $f_{kv}$  increases, and the force  $P_{fw}$  increases accordingly. When the vehicle body moved at a distance equal to the elastic deformation " $e$ " of the braking driving wheels, its further movement is possible only due to the energy applied to the drive wheels from the transmission. If the driven wheels are still braked, the longitudinal deformation of their tires causes an elastic force directed against the movement of the vehicle body. In this case, before the start of sliding or rotation of the driven braking wheels, equation (23) is true.

If the longitudinal movement  $x$  of the vehicle body increases, and the braking force to the driven wheels is greater than the force of their adhesion, which corresponds to equation (27), the driven braking wheels will slip:

$$\frac{M_v}{r_d} > |c_t \cdot (t - x)| > P_{\phi h \max}. \quad (27)$$

$P_{\phi h \max} = R_{zh} \cdot \varphi_x$  is the force of driven wheels adhesion to the road surface ( $\varphi_x$  is the adhesion coefficient).

In this case, the rolling resistance coefficient of the vehicle's driving wheels corresponds to:

$$f_{kv} = \frac{a_{rw}}{r_{dr}} + \frac{M_k}{R_{zw}} \cdot \left( \frac{r_{dr} - r_{kv}}{r_{dr} \cdot r_{kv}} \right) + \frac{G_a \cdot \sin \alpha + P_{\phi h \max}}{R_{zw}}. \quad (28)$$

If during the longitudinal movement of the vehicle body there is a condition:

$$P_{\phi h \max} > c_t \cdot (e - x) > \frac{M_t}{r_{dh}}, \quad (29)$$

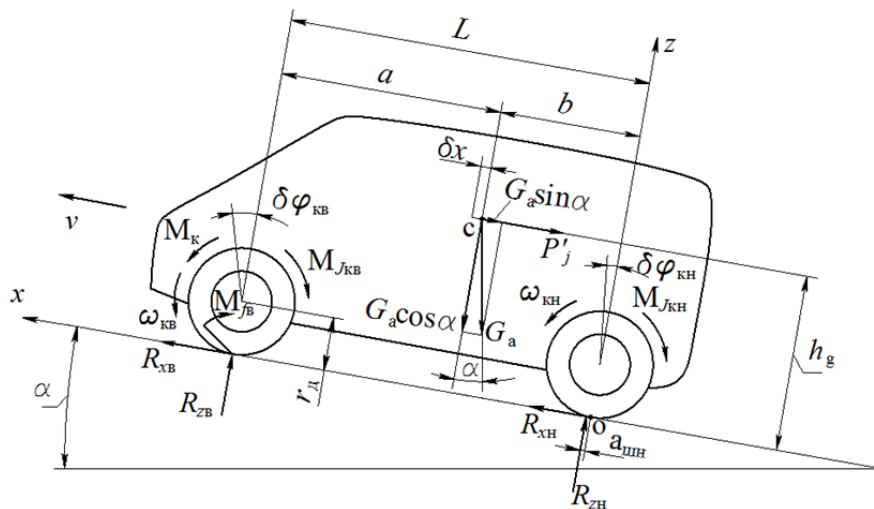


then the driven braking wheels, in addition to the translational motion, will acquire a rotational motion (figure 3). In this case, equation (28) is converted to:

$$f_{kv} = \frac{a_{nv}}{r_{dr}} + \frac{M_k}{R_{zw}} \cdot \left( \frac{r_{dr} - r_{kv}}{r_{dr} \cdot r_{kv}} \right) + \frac{G_a \cdot \sin \alpha}{R_{zw}} + \frac{M_{th} + R_{zh} \cdot a_{th}}{R_{zw} \cdot r_{dh}}, \quad (30)$$

where  $R_{zh} \cdot a_{th} = M_{fh0}$  is the moment of rolling resistance to the driven wheels in the driven mode;  $a_{th}$  is the longitudinal displacement of the normal road reaction to the driven wheels.

Using the obtained results, it is possible, with known methods [14], to compose a mathematical model of the vehicle initial uphill motion with the absence of the drive wheels skidding. Taking into account the false slipping of driving wheels, the vehicle has two degrees of freedom and its movement is described by the equations of the rotational motion of the wheels and the translational motion of the body.



**Figure 3.** Scheme of external influences acting to the vehicle when the torque  $M_k$  applied to the driving wheels and the rotation of the driven wheels.

The system of equations describing the dynamic behavior of the vehicle after applying torque to the drive wheels is:

$$\left. \begin{aligned} \frac{d\omega_{kv}}{dt} &= \frac{M_k - M_{fv}}{J_{kv,red}}; \\ \frac{dv}{dt} &= r_{kv} \cdot \frac{d\omega_{kv}}{dt}; \\ \frac{dx}{dt} &= v. \end{aligned} \right\} \quad (31)$$

where  $J_{kv,red}$  is the reduced moment of inertia to the driving wheels of the vehicle.

The reduced moment of inertia to the drive wheels is the sum of the inertia moment of the rotating engine masses, transmission and drive wheel:

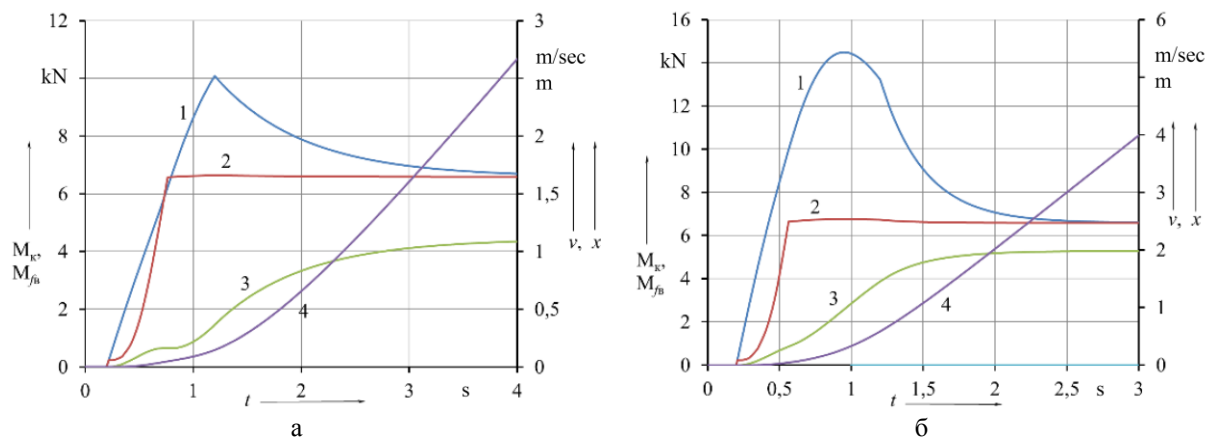
$$J_{kv,red} = J_e \cdot u_{fr}^2 \cdot \eta_{fr} + J_{fr} + \sum J_{kv}. \quad (32)$$

The rolling radius of the drive wheels  $r_{kv}$  provided  $M_k \leq 0,6 \cdot P_{qvw \max} \cdot r_d$  [24] linearly depends on the moment  $M_k$ :

$$r_{kv} = r_d - \lambda \cdot M_k, \quad (33)$$

where  $\lambda = \frac{dr_k}{dM_k}$  is the tires tangential elasticity coefficient of driving axle.

The torque applied to the driving wheels in the first equation of system (31) determines the equation (7), the rolling resistance moment of the driving wheels the equations (10), (11) and in accordance with the movement condition of the equations (19) or (23), (28), (30). In accordance with the proposed mathematical model of the vehicle initial uphill motion, there are obtained parameters for a vehicle with a total weight of 9,500 kg for a 10-degree rise with fuel supply of 15 % and 40 % from the maximum (figure 4).



**Figure 4.** The parameters of the vehicle initial uphill motion with different parameters of engine management: a – low fuel supply; b – high fuel supply; 1 – torque to the driving wheels “ $M_k$ ”; 2 – the moment of resistance to the driving wheels “ $M_{fw}$ ”; 3 – speed “ $v$ ”; 4 – distance “ $x$ ”.

The vehicle dynamic parameters are obtained under the condition that the  $x \leq e$  control of the braking torques to the braking wheels provides the condition:

$$\frac{M_t}{r_d} \geq c_i \cdot (e - x), \quad (34)$$

and for  $x > e$ , the braking torque to the wheels is zero. Figure 4a clearly shows the effect of the tires elastic deformation that were not braked before the initial motion of the non-driving wheels to the dynamics of the vehicle initial uphill motion. If the fuel supply increase (Fig. 4b), the engine applies more energy to the driving wheels, and the tires elastic energy of the braking wheels, all other conditions being the same, remains unchanged, therefore, its influence on the initial movement speed is not so significant.

## 5. Conclusion

The proposed mathematical model of the vehicle initial rectilinear motion on a slope takes into account the tires elastic deformation energy of braking wheels at the beginning of movement. Taking into account the tires deformation energy of braking wheels at different dynamic conditions of the vehicle initial uphill motion, it is possible to estimate more correctly the dynamic and energetic

parameters of the initial movement specify the requirements to the algorithm of automatic vehicle brake control systems and, as a consequence, improve the quality of control process.

The proposed mathematical model of the vehicle initial rectilinear motion on a slope can be used as a theoretical basis for the formation of the structure of automatic systems, analysis and synthesis of control algorithms in this mode

## 6. References

- [1] Gekker F R et al. 1989 *Stsepleniya transportnyih i tyagovyih mashin* [*Clutch transport and traction machines*] (Moscow : Mashinostroenie) p 344 (in Russian)
- [2] Golovchuk A F 2011 Uluchshenie toplivnoy ekonomichnosti i snizhenie dyimnosti traktornyih dizeley putem sovershenstvovaniya sistemyi avtomaticheskogo regulirovaniya [*Improving fuel efficiency and reducing the opacity of tractor diesel engines by improving the automatic control system*] (Harkov : Kharkiv National Automobile and Highway University) p 472
- [3] Zaharik Yu M 2004 Kompleksnyiy zakon upravleniya stsepleniem [Integrated clutch control law] *Avtomobilnaya promyshlennost* [*Automobile Industry*] **9** 23–25 (in Russian)
- [4] Yaryta O A, Mychalevych M G, Leontiev D N, Klymenko V I, Bogomolov V A, Gritsuk I V and Novikova Y B 2018 Features of controlling electropneumatic valves of actuator to control its clutch with acceleration valve *Science & Technique* **17** 64–71, <https://doi.org/10.21122/2227-1031-2018-17-1-64-71>
- [5] Mikhalevich M, Yarita A, Leontiev D, Gritsuk I V et al. 2019 Selection of rational parameters of automated system of robotic transmission clutch control on the basis of simulation modelling *SAE Technical Paper* 2019-01-0029, doi:10.4271/2019-01-0029
- [6] Mychalevich M, Yarita O, Turenko A, Leontiev D, Gritsuk I, Bogomolov V, Klimenko V and Smieszek M 2018 Assessment of operation speed and precision of electropneumatic actuator of mechanical transmission clutch control system *SAE Technical Paper* 2018-01-1295, doi:10.4271/2018-01-1295
- [7] Nghia L 2018 Feedback in automated clutch control circuit for truck start-up process *Science & Technique* **17** 421–431, <https://doi.org/10.21122/2227-1031-2018-17-5-421-431>
- [8] Manish K, Taehyun S and Zhang Y 2007 Shift dynamics and control of dual- clutch transmissions *Mechanism and Machine Theory* **42** 168–182
- [9] Liu F Li Y, Zhang J, Huang H et al. 2002 Robust control for automated clutch of AMT vehicle *SAE Transaction* **111** 2002-01-0933 1360–1367
- [10] Yingchun L and Lifu L 2015 Fuzzy fractional order sliding mode control for automatic clutch of vehicle AMT *International Journal of Smart Home* **9** 53–68, <http://dx.doi.org/10.14257/ijsh.2015.9.2.05>
- [11] Turenko A N and Shuklinov S N 2010 Adaptivnoe tormoznoe upravlenie kolesnyih mashin [Adaptive brake control wheeled vehicles] *Zhurnal avtomobilnyih inzhenerov* **5** 18–21
- [12] Turenko A N, Shuklinov S N and Mihalevich N G 2012 Modelirovanie dinamiki kolesnoy mashinyi s adaptivnyim elektropnevmaticheskim privodom tormozov [Modeling the dynamics of wheeled vehicles with adaptive electro-pneumatic brake drive] *Vestnik Kharkiv National Automobile and Highway University* **56** 66–74
- [13] Tarasik V P 2006 *Teoriya dvizheniya avtomobilya* [*Theory of car movement*] (SPb: BHV-Peterburg) p 478
- [14] Tarasik V P 2017 Matematicheskoe modelirovanie pryamolineynogo dvizheniya avtomobilya [Mathematical modeling of a straight vehicle movement] *Vestnik Belorussko-Rossiyskogo universiteta* **2** 119–130
- [15] Zaharik Yu M and Zaharik An M 2006 Algoritm elektronnoy sistemyi ZS, isklyuchayushey skatyivanie avtomobilya pri troganii na pod'eme [The algorithm of the electronic system ZS, excluding the car rolling off when starting on the rise] *Avtomobilnaya promyshlennost* **2** 14–17
- [16] Shuklinov S N, Gubin A V 2018 Issledovanie rezhimov tormozhenie-stop-nachalo dvizheniya avtomobilya na uklone [Investigation of the braking-stop-start mode of the car on a slope] *Avtomobilnyiy transport* **42** 61–69

- [17] Zhang L and Li H 2006 Hilling start-up control method using accelerometer *Vehicular Electronics and Safety, IEEE International Conference* 276–279
- [18] Taki Y, Yamada N, Nagae A and Ishida Y 2003 Development of brake control system for driving hills applying active wheel speed sensor *JSAE* 34 13–16
- [19] Ge A, Lei Y, Gao Y, Zhang H and Fan J 1998 A research on hill starting control of AMT system of vehicle *Automotive Engineering* 3
- [20] Wang H, Zhao X and Liu H 2009 The electronic-pneumatic hill-starting assist control for heavy-duty vehicles with AMT *Journal of Shandong University: Engineering Science* 5
- [21] Jiang Xuefeng, Xu Xian, Jin Guodong and Chen Wei 2012 Research on hill start control for heavy truck with AMT *The 2nd International Conference on Computer Application and System Modeling* 1169–1172
- [22] Jiang D, Huang Y, Zuo Z and Li H 2017 drivability improvement control for vehicle start-up applied to an automated manual transmission *Hindawi: Shock and Vibration* **2017** 578527
- [23] Jiang M, Zhou J, Chen W and Zhang Y 2011 *Modeling and Simulation of AMT with MWorks* (Liping Chen CAD Center, Huazhong University of Science and Technology, China)
- [24] Litvinov A S, Farobin and Y E 1989 *Avtomobil: Teoriya ekspluatatsionnyih svoystv* [*Car: Theory of operational properties*] Uchebnik dlya vuzov po spetsialnosti «Avtomobili I avtomobilnoe hozyaystvo» (Moscow: Mashinostroenie) p 240
- [25] Kuric I, Mateichyk V, Smieszek M, Tsiuman M et al. 2018 The peculiarities of monitoring road vehicle performance and environmental impact *MATEC Web of Conferences* **244** 030033
- [26] Kuric I, Tlach V, Ságová Z, Cíсар M et al. 2018 Measurement of industrial robot pose repeatability *MATEC Web of Conferences* **244** 01015
- [27] Gritsuk I, Zenkin E Y E, Bulgakov N, Golovan A et al. 2018 The complex application of monitoring and express diagnosing for searching failures on common rail system units *SAE Technical Paper* 2018-01-1773, <https://doi.org/10.4271/2018-01-1773>
- [28] Klets D, Gritsuk I, Makovetskyi A, Bulgakov N et al. 2018 Information security risk management of vehicles *SAE Technical Paper* 2018-01-0015, <https://doi.org/10.4271/2018-01-0015>
- [29] Gritsuk I, Volkov V, Mateichyk V, Grytsuk Y et al. 2018 Information model of V2I system of the vehicle technical condition remote monitoring and control in operation conditions *SAE Technical Paper* 2018-01-0024, <https://doi.org/10.4271/2018-01-0024>
- [30] Volodarets M, Gritsuk I, Chygyryk N, Belousov E et al. 2019 Optimization of vehicle operating conditions by using simulation modeling software *SAE Technical Paper* 2019-01-0099, doi:10.4271/2019-01-0099
- [31] Gritsuk I, Gutarevych Y, Mateichyk V and Volkov V 2016 Improving the processes of preheating and heating after the vehicular engine start by using heating system with phase-transitional thermal accumulator *SAE Technical Paper* 2016-01-0204, <https://doi.org/10.4271/2016-01-0204>