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MAGNETIC FIELD OF CURRENT IN A STRAIGHT CONDUCTOR WITH A RECTANGULAR CROSS-SECTION AND THE ERROR OF ITS DETERMINATION USING THE LINEAR CURRENT MODEL

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The calculation of the magnetic field of currents has important practical and methodological significance. The determination of the magnetic field of current in massive conductors is based on the integration over the entire volume of the conductor of the magnetic fields created by the current elements. It is much easier to calculate the field using the linear current model, but the conditions of its application (the parameters of the conductor cross-section must be much smaller than the distance to the observation point) are idealized and limit the practical application of the model. The literature considers the magnetic field of currents in cylindrical conductors, but in practice, conductors with a rectangular cross-section are used. The purpose of the work is to obtain mathematical expressions for determining the induction of the magnetic field generated by a direct current in a straight long conductor with a rectangular cross-section, on the axes of symmetry of the cross-section, and to calculate the error of applying the linear current model for this, depending on the relative distance to the observation points. For points on the Ox axis (in the direction of the cross-section width), the values of the true magnetic field induction B are greater than the linear current field, and for points on the Ou axis, on the contrary, they are smaller. With an increase in the degree of elongation of the conductor cross-section (decreasing c), the absolute and relative errors of the application of the linear current model increase, since in this case the conductor and its field increasingly differ from a cylindrical conductor, the field of which, like the linear current field, is cylindrically symmetrical. The results of the work have significant practical significance and can be used in the development of electrical devices to determine the true magnetic field, or approximately in the linear current model with a known error. The methodological significance of the work lies in demonstrating the limitations of the application of models in physics and the dependence of the model error on the conditions of its application.

Key words: *magnetic field, conductor with a rectangular cross-section, error of the linear current mode.*

Івашина Ю. К., Заводянний В. В. Магнітне поле струму в прямому провіднику з прямокутним перерізом і похибка його визначення з допомогою моделі лінійного струму

Розрахунок магнітного поля струмів має важливе практичне і методичне значення. Визначення магнітного поля струму в масивних провідниках базується на інтегруванні по всьому об’єму провідника магнітних полів, що створюються елементами струму. Значно простіше розрахувати поле за допомогою моделі лінійного струму, але умови її застосування (параметри перерізу провідника повинні бути значно меншими відстані до точки спостереження) ідеалізовані і обмежують практичне застосування моделі. В літературі *розглядається магнітне поле струмів в циліндричних провідниках, але на практиці використовуються провідники з прямокутним перерізом. Мета роботи – отримати математичні вирази для визначення індукції магнітного поля, породженого постійним струмом в прямому довгому провіднику з прямокутним перерізом, на осях симетрії перерізу, і розрахувати похибку застосування для цього моделі лінійного струму в залежності від відносної відстані до точок спостереження. Для точок на осі Ох (в напрямку ширини перерізу) значення індукції істинного магнітного поля В більші ніж поля лінійного струму, а для точок на осі Оу, навпаки, менші. Із збільшенням ступені витягнутості перерізу провідника (зменшенням с) абсолютна і відносна похибки застосування моделі лінійного струму зростають, так як при цьому провідник і його поле все більше відрізняються від циліндричного провідника, поле якого, як і поле лінійного струму, циліндрично симетричне. Результати роботи мають суттєве практичне значення і можуть бути використаними при розробці електротехнічних пристроїв для визначення істинного магнітного поля, або наближено в моделі лінійного струму з відомою похибкою. Методичне значення роботи полягає в демонстрації обмеженості застосування моделей у фізиці і залежності похибки моделі від умов її застосування.*

Ключові слова: *магнітне поле, провідник з прямокутним перерізом, похибка моделі лінійного струму.*

Introduction. The development and design of various electrotechnical devices require the determination of the magnetic field, which makes the calculation of current magnetic fields of great practical importance. The methodological value of such a calculation lies in demonstrating the use of the linear current model and the dependence of its error on the application conditions.

The determination of the magnetic field of a current in massive conductors is based on integrating the magnetic fields created by current elements throughout the volume of the conductor. This method involves significant mathematical complexity, requires considerable effort, and high qualification. It is much simpler to determine the field using the linear current model, but the conditions for its use (the cross-sectional dimensions of the conductor must be much smaller than the distance to the observation point) are idealized and limit the practical application of this model.

Review of Recent Publications. Textbooks and methodological literature widely describe the calculation of the magnetic field of a current in cylindrical coordinates, based on the Biot–Savart–Laplace law and integration of the fields of current elements [1,2].

The magnetic field of massive conductors is also determined via the vector potential [2,3], which is found by integrating over the conductor's volume. The field is defined as a series of cross-sectional moments, of which the linear current field (zero-order moment) changes the least with distance [3]. This calculation method requires a high level of qualification and deep knowledge.

In practice, rectangular conductors are widely used.

In [4], the magnetic field on the axes of a square conductor and the error of applying the linear current model to its determination were considered. However, such conductors are not used in practice.

We are not aware of any works in which the magnetic field on the symmetry axes of a rectangular conductor and the calculation error using the linear current model are determined.

Research Objective. The goal of this work is to derive mathematical expressions for determining the magnetic field induction generated by a steady current in a long straight conductor with a rectangular cross-section, along the symmetry axes of the cross-section, and to calculate the error of using the linear current model depending on the relative distance to observation points.

Another objective is to determine the effect of the elongation degree of the conductor cross-section on the error of the linear current model.

Presentation of the main material.

Methodology for calculating the magnetic field.

The magnetic field around a conductor with a rectangular cross-section was studied $a \times b$, where a – width, a b – hickness of the conductor. A stationary magnetic field of direct current was considered. It was assumed that the current is distributed evenly over the cross section of the conductor. Current density $j = \frac{J}{ab} = const.$ The conductor is straight and long. $\ell \gg a, b$. The current in the conductor was considered as the sum of the currents in the elementary tubes $dJ = j dS = j\, dxdy$ with a rectangular cross-section, which were considered as linear currents. The magnetic field induction of a direct long tube current was determined based on the Biot-Savart-Laplace law:

$$dB = \frac{\mu_0\, dJ}{2\pi r} = \frac{\mu_0 J\, dxdy}{2\pi a b r}, \tag{1}$$

where r is the distance from the tube to the observation point. The resulting field of all current tubes was determined based on the superposition principle by integrating (2.1) over the entire cross-section of the conductor

$$\overline{B} = \int_S d\overline{B} \tag{2}$$

To integrate a vector $d\overline{B}$, we introduced a coordinate system with the origin at the center of the section and define the projections dB_x i dB_y. Then

$$B_x = \int_S dB_x ; B_y = \int_S dB_y \tag{3}$$

The magnetic field on the axes of symmetry of the section was studied. This is due to the fact that all physical characteristics of objects on the axes of symmetry have extreme values. For example, for a rectangular section on the axis directed along the width, the moment of inertia and stiffness are maximum, and in the direction of the thickness – minimum. In addition, on the axes of symmetry, the integrals (3) have the simplest form.

Magnetic field on the axis of symmetry in the direction of the cross-section width.

Select an arbitrary cross-section element dS_1 and we determine the field induction $d\overline{\mathrm{B}}_{A_1}$, which is created in t.A on the x-axis by the current tube passing through dS_1 according to (1) (Fig. 1). The current is directed in the direction of the z-axis.

Since Ox is the axis of symmetry, it is always possible to find a symmetrical and equal cross-sectional element dS_2, current through which creates a field at point A $dB_{A_2} = d\mathrm{B}_{A_1}$. he components of these vectors in the x-axis direction are compensated, and in the y-axis direction they are added, so the field on the x-axis

$$B_A = \mathrm{B}_{A_y} = \int_S d\mathrm{B}_{A_y} \tag{4}$$

Projection of the field induction of an elementary current tube onto the y axis

$$dB_y = dB \cdot cos\alpha = \frac{\mu_0 J\, dxdy\;(R-x)}{2\pi a b \quad\; r^2},, \tag{5}$$

where R=OA – distance from the center of the section to the observation point,

$$r^2 = (R-x)^2 + y^2, \quad \cos\alpha = \frac{R-x}{r}..$$

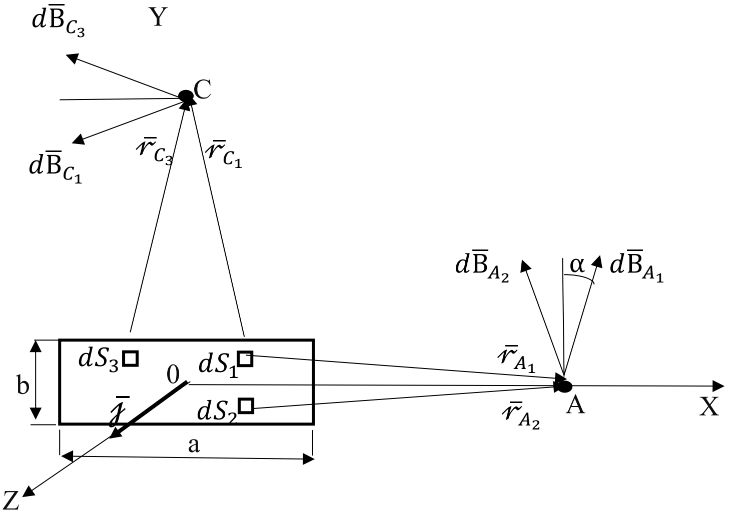

Fig. 1. Magnetic field of current tubes

Magnetic field induction at point A

$$B_A = B_{A_y} = 2\frac{\mu_0 J}{2\,\pi a b}\int_0^{\frac{b}{2}} dy \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{(R-x)dx}{y^2+(R-x)^2} = \frac{\mu_0 J}{\pi a b}\cdot\frac{1}{2}\int_0^{\frac{b}{2}} \ln\frac{y^2+\left(R+\frac{a}{2}\right)^2}{y^2+\left(R-\frac{a}{2}\right)^2}\, dy =$$

$$= \frac{\mu_0 J}{\pi a b}\left(\frac{b}{4}\, ln\frac{\frac{b^2}{4}+\left(R+\frac{a}{2}\right)^2}{\frac{b^2}{4}+\left(R-\frac{a}{2}\right)^2} + \left(R+\frac{a}{2}\right) arctg\frac{b}{2R+a} - \left(R-\frac{a}{2}\right) arctg\frac{b}{2R-a}\right) \qquad (6)$$

Since the field depends on both the distance from the center of the section to the observation point R, and from the cross-sectional dimensions, we introduce the relative distance $\frac{R}{a}$, $b = ca$, where c etermines the degree of elongation of the cross section. Substituting these substitutions into (6) we obtain

$$B_A = \frac{\mu_0 J}{2\pi a}\left[\frac{1}{2}\, ln\frac{4(\frac{R}{a})^2+4\frac{R}{a}+1+c^2}{4(\frac{R}{a})^2-4\frac{R}{a}+1+c^2} + \frac{1}{c}\left[\left(2\frac{R}{a}+1\right) arctg\frac{c}{2\frac{R}{a}+1} - \left(2\frac{R}{a}-1\right) arctg\frac{c}{2\frac{R}{a}-1})\right]\right] \qquad (7)$$

This expression defines the magnetic field induction on the x-axis as a function of the relative distance $\frac{R}{a}$ to the observation point and the degree of elongation of the cross section c. The field on the x-axis is directed perpendicular to this axis.

Magnetic field on the axis of symmetry in the direction of the cross-section thickness

Consider the magnetic field created at point C on the y-axis by two equal and symmetrical current tubes with cross-sections

dS_1 and dS_3. The components of the field induction in the y-axis direction are compensated, and in the x-axis direction they are added, therefore

$$B_{\mathcal{C}} = \mathrm{B}_{\mathcal{C}_x} = 2\frac{\mu_0 \mathcal{J}}{2\,\pi a b}\int_0^{\frac{a}{2}} dx\ \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{(R-y)dy}{x^2+(R-y)^2} = \frac{\mu_0 \mathcal{J}}{\pi a b}\left(\frac{1}{2}\int_0^{\frac{a}{2}} \ln\frac{x^2+\left(R+\frac{b}{2}\right)^2}{x^2+\left(R-\frac{b}{2}\right)^2}\ dx\right) =$$

$$\frac{\mu_0 \mathcal{J}}{2\pi a b}\left(\frac{a}{2}\,ln\frac{\frac{a^2}{4}+\left(R+\frac{b}{2}\right)^2}{\frac{a^2}{4}+\left(R-\frac{b}{2}\right)^2} + (2R+b)arctg\,\frac{a}{2R+b}\ - (2R-b)arctg\,\frac{a}{2R-b}\right) \qquad (8)$$

Let us express the field induction in terms of the relative distance $\frac{R}{a}$ and the degree of elongation of the cross section $c = \frac{b}{a}$

$$B_{\mathcal{C}} = \frac{\mu_0 \mathcal{J}}{2\pi a}\left[\frac{1}{2c}\,ln\frac{4(\frac{R}{a})^2+4\frac{R}{a}+1+c^2}{4(\frac{R}{a})^2-4\frac{R}{a}+1+c^2} + \frac{1}{c}\left[\left(2\frac{R}{a}+c\right)arctg\,\frac{1}{2\frac{R}{a}+c} - \left(2\frac{R}{a}-c\right)arctg\,\frac{1}{2\frac{R}{a}-c})\right]\right] \qquad (9)$$

Vector $\overline{\mathrm{B}}$ at points on the y axis is directed perpendicular to this axis, its modulus is determined by expression (9).

The results of calculating the magnetic field induction depending on the relative distance to the observation points on the axes of symmetry according to (7) and (8) are given in Table 1.

At $\frac{R}{a} \to \infty$ he obtained expressions show that in this case $\mathrm{B} \to 0$, which is a necessary condition of their loyalty.

Error in applying the linear current model to the calculation of the magnetic field of a conductor with a rectangular cross-section

The calculation of the magnetic field based on (7) and (9) is laborious and requires appropriate qualifications. It is much easier to determine the field approximately based on the linear current model. But the condition for using this model (the distance from the observation point is much greater than the characteristics of the conductor cross-section $R \gg a$) idealized and does not allow for practical application of the model. The error of applying the model and its dependence on distance are also unknown.

We have compared the results of calculating the true magnetic field of a direct current in a long straight conductor with a rectangular cross-section based on (7) and (9) with the field of a linear current of the same magnitude passing through the center of the cross-section. The field of a linear current is determined by.

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$$B_{л} = \frac{\mu_0 \mathcal{J}}{2\,\pi R} = \frac{\mu_0 \mathcal{J}}{2\,\pi a\left(\frac{R}{a}\right)} \qquad (10)$$

In this expression, the field is determined depending on the relative distance $\frac{R}{a}$.

Table 1

Dependence of the magnetic field induction B, created by a direct current in a long straight conductor with a rectangular cross-section, on the field induction of a linear current $B_л$, absolute error, relative error from the relative distance to the observation points on the axis of symmetry of the section

Parameter	**Relative distance, $\frac{R}{a}$**				
	0,5	**1**	**2**	**4**	**6**
on the OX axis, C=0,2					
$B, \frac{\mu_0 I}{2\pi a}$	3,6456	1,0892	0,5158	0,2527	0,1673
$B_л, \frac{\mu_0 I}{2\pi a}$	2,000	1,000	0,500	0,2500	0,1667
$\Delta B, \frac{\mu_0 I}{2\pi a}$	1,6486	0,0892	0,0158	0,0027	0,0006
$\frac{\lvert\Delta B\rvert}{B}$, %	45,3	8,2	3,1	1,1	0,4
on the OY axis, C=0,2					
$B, \frac{\mu_0 I}{2\pi a}$	1,5773	0,9332	0,4774	0,2455	0,166
$B_л, \frac{\mu_0 I}{2\pi a}$	2,000	1,000	0,500	0,2500	0,1667
$\Delta B, \frac{\mu_0 I}{2\pi a}$	-0,4227	-0,0668	-0,0226	-0,0045	-0,0007
$\frac{\lvert\Delta B\rvert}{B}$, %	26,8	7,2	3,9	1,8	0,4
on the OX axis, C=0,5					
$B, \frac{\mu_0 I}{2\pi a}$	2,3966	1,0647	0,5081	0,2518	0,1671
$B_л, \frac{\mu_0 I}{2\pi a}$	2,000	1,000	0,500	0,2500	0,1667
$\Delta B, \frac{\mu_0 I}{2\pi a}$	0,3966	0,0647	0,0081	0,0012	0,0004
$\frac{\lvert\Delta B\rvert}{B}$, %	16,6	6,1	1,6	0,5	0,2
on the OY axis, C=0,5					
$B, \frac{\mu_0 I}{2\pi a}$	1,6045	0,9408	0,4896	0,2485	0,1662
$B_л, \frac{\mu_0 I}{2\pi a}$	2,000	1,000	0,500	0,2500	0,1667
$\Delta B, \frac{\mu_0 I}{2\pi a}$	-0,3905	-0,0592	-0,0104	-0,0015	-0,0005
$\frac{\lvert\Delta B\rvert}{B}$, %	24,2	6,3	2,1	0,6	0,3

The absolute error of the model application was calculated $\Delta B = B - B_{л}$ and relative error $\mathcal{E} = \frac{|\Delta B|}{B}$ depending on the relative distance to the observation points on the axes of symmetry of the section. To determine the effect of the elongation of the section, calculations were carried out for $c = \frac{b}{a} = 0{,}2$ and $c = 0{,}5$. The calculation results are presented in Table 1.

Analysis of the calculation results given in the table and Fig. 2 shows that when approaching the conductor, the error of using the linear current model to calculate the magnetic field on the axes of symmetry of the DC cross-section passing through a long straight conductor with a rectangular cross-section increases sharply.

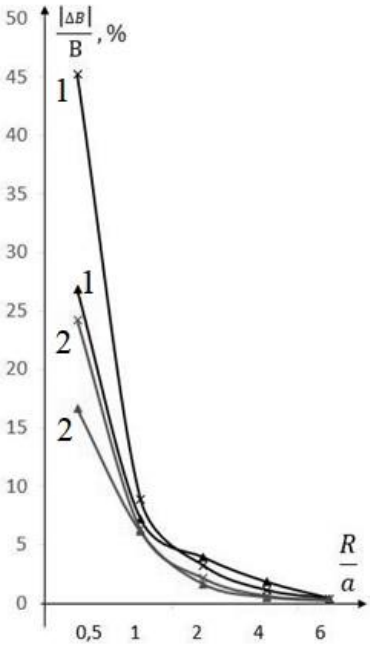

Fig. 2. Dependence of the relative error of determining the magnetic field induction on the axes of symmetry of a straight conductor with a rectangular cross-section on the relative distance to the observation point: × *– on the OX axis,* ▲ *– on the OY axis,*
▬ *– for C- 0,2;* ▭ *– for C=0,5*

With increasing relative distance, the error decreases and at $\frac{R}{a} \geq 6$ becomes less than 0.4% for all cases. For points on the Ox axis (in the direction of the cross-sectional width), the values of the true magnetic field induction B are greater than the linear current field, and for points on the Ouy axis, on the contrary, they are smaller. For these points, the absolute deviation ΔB is negative. With an increase in the degree of elongation of the conductor cross-section (decreasing c), the absolute and relative errors of the application of the linear current model increase, since in this case the conductor and its field increasingly differ from a cylindrical conductor, the field of which, like the linear current field, is cylindrically symmetrical.

The obtained error values make it possible to fairly simply determine the magnetic field of a long straight conductor with a rectangular cross-section using the linear current model approximately with a known error, which opens the way to the practical application of this model.

Conclusions and prospects for further research. Mathematical expressions of the dependence of the induction of a true magnetic field of direct current in a long straight conductor with a rectangular cross-section on the relative distance to the observation point on the axes of symmetry of the cross-section, where the field induction takes extreme values, have been obtained. Their validity is confirmed by the fact that at a large distance, when $\frac{R}{a} \to \infty$, the field induction tends to zero. On the axis in the direction of the cross-sectional width, the true field is stronger than the field of the same linear current, and in the direction of the thickness – on the contrary, less. On the axes of symmetry, the vector is perpendicular to the axes.

The error of applying the linear current model to the calculation of the current field in a conductor with a rectangular cross-section has been calculated depending on the relative distance to the observation points on the axes of symmetry of the cross-section, where it has extreme values.

The results of the work have significant practical significance and can be used in the development of electrical devices to determine the true magnetic field, or approximated in the linear current model with a known error, which opens the way to the practical application of the model.

The methodological significance of the work lies in demonstrating the limitations of the application of physical models and the dependence of their error on the conditions of use of the model.

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